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## Public-good valuation and intrafamily allocation

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#### Abstract

I derive the value of marginal changes in a public good for two-person households, measured alternatively by household member i's willingness to pay (WTP) for the good on behalf of the household, WTPi(H), or by the sum of individual WTP values across family members, WTP(C). Households are assumed to allocate their resources in efficient Nash bargains over one private good for each member, and one common household good. WTPi $(\mathrm{H})$ is then found by trading off the public good against the household good, and WTP(C) by trading the public good off against the private goods. I show that $\mathrm{WTPi}(\mathrm{H})$ is on average a correct representation of WTP(C), but is higher (lower) than this average when member 1 has a higher (lower) marginal public good value than member 2. Pure and paternalistic altruism (the latter attached to consumption of the public good) both move each member's WTP on behalf of the household closer to the true aggregate WTP, while only the latter raises aggregate WTP. The results have important implications for interpretation of results from contingent valuation surveys of public-goods. In a large sample, individuals tend to represent households correctly on average when asked about household WTP, and counting all members' WTP answers on behalf of the household will lead to double counting.


## 1. Introduction

In contingent valuation (CV) surveys, questions eliciting respondents' willingness to pay (WTP) for (small) increases in public goods provision are typically phrased in the following two alternative ways: ${ }^{1}$ "How much are you willing to pay, on behalf of your household, for a (small) increase in the quantity of a public good?"; or: "How much are you, personally, willing to pay for a (small) increase in the quantity of a public good?". A main purpose of this paper is to study the relationship between the answers to these two questions, in the context of a simple model of household behavior. A main result from the paper is that an individual's true WTP on behalf of the household is identical to the sum of household members' personal WTP levels, if and only if the two household members have the same marginal valuation of the pure public good in terms of the household good. When members value the public good differently, a given member over (under-) values the public good on behalf of the household when his or her value is higher (lower) than that of the other member. On average these values are however correct. In a large sample of respondents, individual household members will then represent the entire household correctly.

These results are derived within a model where each household consists of two members, each with (cardinal and selfish) preferences over three distinct goods: one private good, one household good consumed commonly within the household only, and one pure public good (consumed by all in society) provided outside of the household. The household has a given budget and determines its allocation in an efficient asymmetric Nash bargain. This solution yields a lower marginal utility of consumption for the household good than for the private good, with greater disparity for members with lower bargaining strength. The true aggregate household WTP for
the public good can be represented by the sum of individual private WTP levels for the public good, in terms of each member's own private good. Member i's WTP for the public good on behalf of the household is instead derived considering this member's willingness to give up units of the household good.

In section 3 I show that these basic results hold also under "pure" altruism among family members, where the utility level of each member enters into the utility function of the other. Given that both members are to pay their marginal valuations, overall valuations are not increased by altruism. Individual valuations on behalf of the household are however closer to true aggregate household valuation whenever they differ. In section 4 I instead consider "paternalistic" altruism where a given household member cares about the other member's consumption of the public good. Then greater altruism implies that public-good valuations generally increase. Altruism also here makes one member's WTP on behalf of the household more equal to the sum of individual WTP levels, much in the same way as under pure altruism. Section 5 extends the analysis in the non-altruistic case to more general utility functions where the three goods are no longer strongly separable in consumption. I show that the main conclusions from section 2 (in particular, that the individual's expressed valuation on behalf of the household is always correct on the average) then still hold.

This paper contributes to a resolution of a long-standing controversy and confusion in the CV literature, by showing that one household member tends to represent the household correctly given that households bargain efficiently over private and household goods. Although ignored by many family economists (such as Becker (1991)), several contributions on intrafamily allocation have considered efficient

[^0]bargaining among family members. ${ }^{2}$ The paper most closely related to this is perhaps Quiggin (1998) who studies household versus individual household WTP assuming that a purely private good is traded off only against a public good. ${ }^{3}$ He then shows that high degrees of altruism are necessary to make individual and household valuations similar. In my model altruism is not essential for this conclusion.

## 2. The basic model with no altruism

Consider a family with two members, with an exogenous common household budget R (net of taxes), shared among purely private consumption $\mathrm{C}_{\mathrm{i}}$ for members $\mathrm{i}=$ 1,2 , and a household good consumed jointly by members in the amount $\mathrm{H} .{ }^{4}$ The family budget constraint is then simply $\mathrm{R}=\mathrm{C}_{1}+\mathrm{C}_{2}+\mathrm{H}$. The two members bargain over $\mathrm{C}_{1}, \mathrm{C}_{2}$ and H in an asymmetric Nash bargain with relative bargaining strengths $\beta$ and $1-\beta .{ }^{5}$ We initially disregard altruism, i.e., no element of other individuals' utility or consumption enters into the utility function of a given member. Member i has a utility function which is strongly separable in its three arguments, of the form

$$
\begin{equation*}
U_{i}=u_{i}\left(C_{i}\right)+v_{i}(H)+z_{i}(P), i=1,2, \tag{1}
\end{equation*}
$$

[^1]where the $u_{i}, v_{i}$ and $z_{i}$ can be viewed as cardinal (von Neumann-Morgenstern) utility functions. They are increasing and strictly concave and fulfil standard Inada conditions, i.e., $\mathrm{f}^{\prime}>0, \mathrm{f}^{\prime \prime}<0$ and $\lim (\mathrm{A} \rightarrow 0) \mathrm{f}^{\prime}=\infty, \lim (\mathrm{A} \rightarrow \infty) \mathrm{f}^{\prime}=0$, for $\mathrm{f}=\mathrm{u}, \mathrm{v}$ and z , and $\mathrm{A}=\mathrm{C}, \mathrm{H}$ and P , respectively. The Nash product will be expressed as
\[

$$
\begin{equation*}
N P(1)=\left[u_{1}\left(C_{1}\right)+v_{1}(H)\right]^{\beta}\left[u_{2}\left(C_{2}\right)+v_{2}(H)\right]^{1-\beta} . \tag{2}
\end{equation*}
$$

\]

To derive the Nash bargaining solution, maximize the Lagrangian $L(1)=N P(1)-\lambda\left(C_{1}\right.$ $+\mathrm{C}_{2}+\mathrm{H}-\mathrm{R}$ ) with respect to $\mathrm{C}_{1}, \mathrm{C}_{2}$ and H , to yield the first-order conditions ${ }^{6}$

$$
\begin{equation*}
\frac{\partial L(1)}{\partial C_{1}}=\beta N_{1}^{\beta-1} N_{2}^{1-\beta} u_{1}^{\prime}\left(C_{1}\right)-\lambda=0 \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial L(1)}{\partial C_{2}}=(1-\beta) N_{1}^{\beta} N_{2}^{-\beta} u_{2}^{\prime}\left(C_{2}\right)-\lambda=0 \tag{4}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial L(1)}{\partial H}=\beta N_{1}^{\beta-1} N_{2}^{1-\beta} v_{1}^{\prime}(H)+(1-\beta) N_{1}^{\beta} N_{2}^{-\beta} v_{2}^{\prime}(H)-\lambda=0 . \tag{5}
\end{equation*}
$$

Here the $\mathrm{N}_{\mathrm{i}}$ are the Nash maximands (i.e., expressions inside the respective square brackets in (2)). Eliminating $\lambda$ implies

$$
\begin{equation*}
\frac{1}{1+n} u_{1}^{\prime}\left(C_{1}\right)=\frac{n}{1+n} u_{2}^{\prime}\left(C_{2}\right)=v^{\prime}(H) \tag{6}
\end{equation*}
$$

[^2]where $n=[(1-\beta) / \beta]\left(N_{1} / N_{2}\right)$. $n$ may be viewed as a "primitive" of the bargaining solution, and where we have used the normalization $v_{1}{ }^{\prime}(H)=v_{2}{ }^{\prime}(H)=v^{\prime}(H)$, for any equilibrium value of $H .^{7}$ When $n \rightarrow 0$, only member 1 has bargaining power; when $n=$ 1, both members have the same "effective bargaining power" (as under identical utility functions and $\beta=1 / 2$ ); and when $n \rightarrow \infty$, only member 2 has bargaining power. (6)-(7) give the marginal rates of substitution between the private goods $\mathrm{C}_{\mathrm{i}}$ and the family good H , under an efficient Nash bargaining solution. It is a standard (Samuelsonian) solution whereby the marginal value of the household ("local public") good equals a (weighted) sum of marginal private-good values. ${ }^{8}$ Generally, when $\beta \in$ ( 0,1 ) (and thus $\mathrm{n}>0$ ), $\mathrm{u}_{\mathrm{i}}{ }^{\prime}\left(\mathrm{C}_{\mathrm{i}}\right)$ exceeds $\mathrm{v}_{\mathrm{i}}{ }^{\prime}(\mathrm{H})$. When R increases marginally, the amount of household goods H or private goods $\mathrm{C}_{\mathrm{i}}$ increase. In the latter case household member 1 (2) however only receives a fraction $1 /(1+n)(n /(1+n))$ to spend on increased $\mathrm{C}_{\mathrm{i}}$. At an optimum the consumption value of increased $\mathrm{C}_{\mathrm{i}}$ must equal the value of the increase in H . At an optimal solution the marginal utility of personal consumption must then exceed that of common consumption.

Face now member i with the question: "What is the highest payment you are willing to make, on behalf of your household, in return for a (small) increase in the public good P ; i.e., what, in your view, is your household's maximum willingness to pay (WTP) for such an increase in the public good?" For "small" changes this is the same as asking how many units of H member i is willing to forsake, to obtain this increase in P . We find this value to be

[^3]\[

$$
\begin{equation*}
W T P_{i}(H)=-\frac{d H}{d P}\left(U_{i}=\text { const. }\right)=\frac{z_{i}^{\prime}(P)}{v^{\prime}(H)}, i=1,2 . \tag{8}
\end{equation*}
$$

\]

$\mathrm{WTP}_{\mathrm{i}}(\mathrm{H})$ is here interpreted as household member i's WTP for the public good on behalf of the household, i.e. in terms of H .

Denote the maximum willingness to pay for the public good in terms of the private good for household member i by $\mathrm{WTP}_{\mathrm{i}}(\mathrm{C})$, and the sum of the two by $\mathrm{WTP}(\mathrm{C}) . \mathrm{WTP}_{\mathrm{i}}(\mathrm{C})$ is the correct answer to the question: "How much are you willing to give up, in terms $\mathrm{C}_{\mathrm{i}}$, for a small increase in P?" We find

$$
\begin{equation*}
W T P_{i}(C)=-\frac{d C_{i}}{d P}\left(U_{i}=\text { const. }\right)=\frac{z_{i}^{\prime}(P)}{u_{i}^{\prime}\left(C_{i}\right)}, i=1,2 . \tag{9}
\end{equation*}
$$

From (6)-(7), $\mathrm{WTP}_{i}(\mathrm{C})<\mathrm{WTP}_{\mathrm{i}}(\mathrm{H})$ since $\mathrm{u}_{\mathrm{i}}{ }^{\prime}(\mathrm{C})>\mathrm{v}^{\prime}(\mathrm{H})$. We then have

$$
\begin{equation*}
W T P(C)=\frac{z_{1}^{\prime}(P)}{u_{1}^{\prime}\left(C_{1}\right)}+\frac{z_{2}^{\prime}(P)}{u_{2}\left(C_{2}\right)}=\frac{z_{1}^{\prime}(P)+n z_{2}^{\prime}(P)}{(1+n) v^{\prime}(H)}, \tag{10}
\end{equation*}
$$

where the latter equality is found using (6)-(7). The ratio of $\mathrm{WTP}_{1}(\mathrm{H})$ to $\mathrm{WTP}(\mathrm{C})$, denoted $\mathrm{R}(1)$, is then given by

$$
\begin{equation*}
R(1)=\frac{W T P_{1}(H)}{W T P(C)}=\frac{(1+n) z_{1}(P)}{z_{1}(P)+n z_{2}(P)} . \tag{11}
\end{equation*}
$$

(11) leads to the following result.

[^4]Proposition 1: Assume Nash bargaining over private and household goods, and no altruism. Then WTP for a public good, as expressed by household member 1 on behalf of the household, is greater (smaller) than the sum of the two household members' private WTP for the good, if and only if member I's marginal valuation of the public good in terms of the common household good is higher (lower) than that of member 2.

Proposition 1 implies that if and only if the two family members have the same marginal value of the public good in terms of the household good, any one of them expresses household WTP correctly. Perhaps surprisingly, this result is independent of altruism and of the relative bargaining powers of family members. It only depends on efficient Nash bargaining over private and household goods within the household. The result also implies that aggregating up individual WTP values stated on behalf of the household, across household members, always leads to double counting. This contrasts other work, in particular Jones-Lee (1992) and Quiggin (1998), where such double counting follows from altruistic preferences.

Certain generalizations are straightforward. First, with $\mathrm{m}>2$ household members and a symmetric solution where each member has the same bargaining parameter $1 / \mathrm{m}$, the sum of private WTP still equals one member's WTP in terms of the household good given that members' marginal valuations of the pure public good are identical. Secondly, in section 5 below I generalize to a more complex utility function where the three goods in question may be complements and substitutes in consumption, and still find quite similar results. Thirdly, while the model framework above is cardinal, most of the results readily generalize to an ordinal one where we only require efficient

[^5]household resource allocation over private and household goods. The main difference is that the bargaining parameter n would then be indeterminate. Other main results (that one household member always represents the household correctly given that marginal rates of substitution between the pure public good and the household good are the same for both members; and that one member "on average" always represents the household correctly) will however still hold.

## 3. Pure intra-household altruism

In this and the next section I generalize the above model in two directions, to take into consideration the possibility of altruistic preferences, of two different types. In this section I consider the case of only "pure" (nonpaternalistic) intrafamily altruism. ${ }^{9}$ Each household member now attaches utility to the general utility level enjoyed by the other member. The utility of household member i can then be expressed as

$$
\begin{equation*}
U_{i}=u_{i}\left(C_{i}\right)+v_{i}(H)+z_{i}(P)+\alpha_{i}\left[u_{j}\left(C_{j}\right)+v_{j}(H)+z_{j}(P)\right], i, j=1,2, i \neq j \tag{12}
\end{equation*}
$$

Here $\alpha_{i} \in(0,1)$ is a relative weight attached by member $i$ to member $j$, relative to the weight attached to oneself. $\alpha_{i}=1$ represents what may be denoted complete altruism, which we rule out except as a possible limit case. We may have $\alpha_{1} \neq \alpha_{2}$ whereby members have different degrees of altruism. The Nash product is now

$$
\begin{align*}
& N P(2)=\left[u_{1}\left(C_{1}\right)+v_{1}(H)+\alpha_{1}\left(u_{2}\left(C_{2}\right)+v_{2}(H)\right)\right]^{\beta}  \tag{13}\\
& {\left[u_{2}\left(C_{2}\right)+v_{2}(H)+\alpha_{2}\left(u_{1}\left(C_{1}\right)+v_{1}(H)\right)\right]^{1-\beta}}
\end{align*}
$$

[^6]The Lagrangian $\mathrm{L}(2)$ is now formed in a way corresponding to $\mathrm{L}(1)$, and maximized with respect to the $\mathrm{C}_{\mathrm{i}}$ and H under the budget constraint to yield

$$
\begin{equation*}
\frac{\partial L(2)}{\partial C_{1}}=\left[\beta N_{1}^{\beta-1} N_{2}^{1-\beta}+\alpha_{2}(1-\beta) N_{1}^{\beta} N_{2}^{-\beta}\right] u_{1}^{\prime}\left(C_{1}\right)-\lambda=0 \tag{14}
\end{equation*}
$$

$$
\begin{align*}
& \frac{\partial L(2)}{\partial C_{2}}=\left[(1-\beta) N_{1}^{\beta} N_{2}^{-\beta}+\alpha_{1} \beta N_{1}^{\beta-1} N_{2}^{1-\beta}\right] \mu_{2}^{\prime}\left(C_{2}\right)-\lambda=0  \tag{15}\\
& \quad \frac{\partial L(2)}{\partial H}=\beta N_{1}^{\beta-1} N_{2}^{1-\beta}\left[v_{1}^{\prime}(H)+\alpha_{1} v_{2}^{\prime}(H)\right] \\
& \quad+(1-\beta) N_{1}^{\beta} N_{2}^{-\beta}\left[v_{2}^{\prime}(H)+\alpha_{2} v_{1}^{\prime}(H)\right]-\lambda=0
\end{align*}
$$

Again using the normalization $\mathrm{v}_{1}{ }^{\prime}(\mathrm{H})=\mathrm{v}_{2}{ }^{\prime}(\mathrm{H})=\mathrm{v}^{\prime}(\mathrm{H})$, we find the first-order conditions

$$
\begin{equation*}
\left(1+\alpha_{2} n\right) u_{1}^{\prime}\left(C_{\mid}\right)=\left(\alpha_{1}+n\right) u_{2}^{\prime}\left(C_{2}\right)=\left[1+\alpha_{1}+n\left(1+\alpha_{2}\right)\right] v^{\prime}(H) . \tag{17}
\end{equation*}
$$

The new parameters $\alpha_{\mathrm{i}}$ here affect the intrafamily resource allocation, relative to the conditions (6)-(7) under no altruism. A marginal increase in P now has total utility effect $\mathrm{z}_{\mathrm{i}}{ }^{\prime}(\mathrm{P})+\alpha_{\mathrm{i}} \mathrm{z}_{\mathrm{j}}{ }^{\prime}(\mathrm{P})$ for household member $\mathrm{i}(\neq \mathrm{j})$. Differentiating (12) with respect to H and P we find member i's WTP on behalf of the household as

$$
\begin{equation*}
W T P_{1}^{N}(H)=-\frac{d H}{d P}=\frac{1}{1+\alpha_{i}} \frac{z_{i}^{\prime}(P)+\alpha_{i} z_{j}^{\prime}(P)}{v^{\prime}(H)}, i, j=1,2, i \neq j . \tag{19}
\end{equation*}
$$

[^7]To derive private valuations in this case, it now matters exactly how the valuation question is posed to the respondent. Consider the following two possibilities:

1. What is your private WTP for an extra unit of the public good, in terms of reduced consumption of your own private consumption good, given that you only, and not the other family member, is to pay for this good?
2. What is your private WTP for an extra unit of the public good, in terms of reduced consumption of your own private consumption good, when also the other family member pays his or her own private WTP for the good?

Given that WTP for the public good is elicited from one family member, and only this member is to pay, the first interpretation may appear reasonable. WTP values, denoted $\mathrm{WTP}_{\mathrm{i}}{ }^{\mathrm{N}}(\mathrm{C})$, can then be found differentiating (12) with respect to $\mathrm{C}_{\mathrm{i}}$ and Z (and holding $\mathrm{C}_{\mathrm{j}}$ and H constant):

$$
\begin{equation*}
W T P_{i}^{N}(C)=-\frac{d C_{i}}{d P}=\frac{z_{i}^{\prime}(P)+\alpha_{i} z_{j}^{\prime}(P)}{u_{i}^{\prime}\left(C_{i}\right)}, i, j=1,2, i \neq j . \tag{20}
\end{equation*}
$$

Aggregate household WTP is then found aggregating (20) over i, as

$$
\begin{equation*}
W T P^{N}(C)=\frac{z_{1}{ }^{\prime}(P)+\alpha_{1} z_{2}{ }^{\prime}(P)}{u_{1}{ }^{\prime}\left(C_{1}\right)}+\frac{z_{2}{ }^{\prime}(P)+\alpha_{2} z_{1}{ }^{\prime}(P)}{u_{2}{ }^{\prime}\left(C_{2}\right)} . \tag{21}
\end{equation*}
$$

When instead all members of society are required to pay for increases in the public good, question alternative 2 is more relevant. ${ }^{10}$ The WTP measure can then be derived from the equation set

$$
\begin{align*}
& u_{1}^{\prime}\left(C_{1}\right) d C_{1}+\alpha_{1} u_{2}^{\prime}\left(C_{2}\right) d C_{2}=-\left(z_{1}^{\prime}(P)+\alpha_{1} z_{2}^{\prime}(P)\right) d P  \tag{22}\\
& u_{2}^{\prime}\left(C_{2}\right) d C_{2}+\alpha_{2} u_{1}^{\prime}\left(C_{1}\right) d C_{1}=-\left(z_{2}^{\prime}(P)+\alpha_{2} z_{1}^{\prime}(P)\right) d P . \tag{23}
\end{align*}
$$

Solving (22)-(23) simultaneously for $\mathrm{dC}_{1}$ and $\mathrm{dC}_{2}$ in terms of dP yields (9), as under purely selfish preferences. Aggregate household WTP then equals WTP(C) from (10), where $\mathrm{WTP}(\mathrm{C})<\mathrm{WTP}^{\mathrm{N}}(\mathrm{C}) .{ }^{11}$ Intuitively, when member 2 pays, member 1's utility is reduced due to altruistic concerns about member 2's reduced personal consumption.

WTP(C) will in the following be viewed as the true measure of household WTP for a small increase in P , as it involves simultaneous changes in $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ for the two household members such that both utilities are kept constant. We then have:

Proposition 2: Assume that the analysis in section 2 applies except that household members exhibit pure altruism as defined. Aggregate WTP for the public good is then invariant to a change in the degree of altruism.

Compare now individual WTP on behalf of the household, to the "true" sum of individual WTP levels, WTP(C). Using (17)-(18), WTP(C) can be expressed as

[^8]\[

$$
\begin{equation*}
W T P(C)=\frac{\left(1+\alpha_{2} n\right) z_{1}^{\prime}(P)+\left(\alpha_{1}+n\right) z_{2}^{\prime}(P)}{1+\alpha_{1}+n\left(1+\alpha_{2}\right)} \frac{1}{v^{\prime}(H)} . \tag{24}
\end{equation*}
$$

\]

The ratio of $\mathrm{WTP}_{1}{ }^{\mathrm{N}}(\mathrm{H})$ to $\mathrm{WTP}(\mathrm{C})$, denoted $\mathrm{R}(2)$, is
(25) $R(2)=\frac{W T P_{1}^{N}(H)}{W T P(C)}=\frac{1+\alpha_{1}+n\left(1+\alpha_{2}\right)}{1+\alpha_{1}} \frac{z_{1}{ }^{\prime}(P)+\alpha_{1} z_{2}{ }^{\prime}(P)}{\left(1+\alpha_{2} n\right) z_{1}{ }^{\prime}(P)+\left(\alpha_{1}+n\right) z_{2}{ }^{\prime}(P)}$.

We can now demonstrate the following result:

Proposition 3: Assume that marginal valuations of the public good differ. Then one individual's expressed marginal WTP for the public good, on behalf of the household, is closer to correct aggregate household WTP, the greater the degree of pure altruism for either member.

This result is found differentiating $R(2)$ with respect to the $\alpha_{i}$. Altruism here leads to averaging of marginal valuations across household members. The closer $\alpha_{1}$ and $\alpha_{2}$ are to one (the "complete altruism" case) the closer $R(2)$ is to one when $z_{1}{ }^{\prime}(P) \neq \mathrm{z}_{2}{ }^{\prime}(\mathrm{P})$.

Under pure altruism, individual expressed WTP on behalf of the household is always moved in the direction of true aggregate WTP, which is generally unaltered.

## 4. Paternalistic altruism with respect to the public good

In this section I consider "paternalistic" altruism, where the utility of each member depends on the other member's consumption of the public good but no other goods. ${ }^{12}$ This can be relevant e.g. when P is a cultural or educational good and members care about each other's "cultivation" or education level; or when P is health or environmental goods and members care about each other's health state and longevity. The underlying motivation for the "altruistic" member may then be at least partly selfish. ${ }^{13}$ Now the utility of member $i$ is expressed as ${ }^{14}$

$$
\begin{equation*}
U_{i}=u_{i}\left(C_{i}\right)+v_{i}(H)+z_{i}(P)+\alpha_{i} z_{j}(P), i, j=1,2, i \neq j . \tag{26}
\end{equation*}
$$

The intrafamily resource allocation is unaltered by such altruism given that payments for the public good do not change. Thus (3)-(7) still describe this allocation. Member i's WTP on behalf of the household is now given by

$$
\begin{equation*}
W T P_{i}^{A}(H)=-\frac{d H}{d P}\left(U_{i}=\text { const. } .\right)=\frac{z_{i}^{\prime}(P)+\alpha_{i} z_{j}^{\prime}(P)}{v^{\prime}(H)}, i, j,=1,2, i \neq j . \tag{27}
\end{equation*}
$$

[^9]The expression for aggregate household WTP, as the sum of individual values (in terms of the private good), is here $\mathrm{WTP}^{\mathrm{N}}(\mathrm{C})$ from (21), as under pure altruism when question 1 in section 3 is evoked. Since $\operatorname{WTP}^{\mathrm{N}}(\mathrm{C})>\mathrm{WTP}(\mathrm{C})$, and this disparity increases in the $\alpha_{i}$, we have

Proposition 4: Assume that each household member's altruism is paternalistic with respect to the opposite member's public-good consumption. Then household WTP for the public good is increased by higher degrees of altruism.

The ratio $\mathrm{WTP}_{1}{ }^{\mathrm{A}}(\mathrm{H})$ to $\mathrm{WTP}^{\mathrm{N}}(\mathrm{C})$, denoted $\mathrm{R}(3)$, given by

$$
\begin{equation*}
R(3)=\frac{W T P_{1}^{A}(H)}{W T P^{N}(C)}=\frac{(1+n)\left(z_{1}{ }^{\prime}(P)+\alpha_{1} z_{2}{ }^{\prime}(P)\right)}{z_{1}{ }^{\prime}(P)+\alpha_{1} z_{2}{ }^{\prime}(P)+n\left(z_{2}{ }^{\prime}(P)+\alpha_{2} z_{1}{ }^{\prime}(P)\right)} . \tag{28}
\end{equation*}
$$

From (28), member 1's WTP for the public good on behalf of the household equals true aggregate household WTP in the special case of $\left(1-\alpha_{1}\right) z_{2}{ }^{\prime}(P)=\left(1-\alpha_{2}\right) z_{1}{ }^{\prime}(P)$. One also easily finds, from (28), that when $\alpha_{1}=\alpha_{2}=\alpha$, an increase in $\alpha$ moves $R(3)$ closer to unity whenever $\mathrm{z}_{1}{ }^{\prime}(\mathrm{P}) \neq \mathrm{z}_{2}{ }^{\prime}(\mathrm{P})$; when $\alpha_{2}=0, \mathrm{R}(3)$ increases uniformly in $\alpha_{1}$; and when $\alpha_{1}=0, R(3)$ is reduced uniformly in $\alpha_{2}$. A proportional increase in the degree of mutual altruism always makes one member's valuation on behalf of the household more equal to true aggregate valuation, as under pure altruism in section 3 . Now, however, when only the valuing member is altruistic, increased altruism increases that member's valuation on behalf of the household, relative to true aggregate valuation. The opposite happens when the other member only is altruistic, and altruism increases. These cases are intuitively obvious. To take the last case,
increased altruism does not change member 1's valuation on behalf of the household, but increases true aggregate valuation, and $R(3)$ drops.

## 5. More general preference relationship in the non-altruistic case

A possible weakness of the formulation in the model above is the assumption that preferences are assumed to be strongly separable, between the three goods $\mathrm{C}, \mathrm{H}$ and P. I will now consider the main implications of more general preference relationships for the two household members, over the goods $\mathrm{C}, \mathrm{H}$ and P . For simplicity (and without much loss of generality) I now revert to the case of no altruism, dealt with in section 2 above. Assume that

$$
\begin{equation*}
U_{i}=U_{i}\left(C_{i}, H, P\right), \quad i=1,2, \tag{29}
\end{equation*}
$$

where $U_{i}$ is a standard (vNM) utility function, assumed strictly concave in $C_{i}, H$ and P, which may generally differ between the two individuals. The Nash bargaining solution now has the same basic setup as before. The Nash maximands entering into the bargaining solution will here be specified as $\mathrm{N}_{\mathrm{i}}=\mathrm{U}_{\mathrm{i}}\left(\mathrm{C}_{\mathrm{i}}, H, \mathrm{P}\right)-\mathrm{U}_{\mathrm{i}}(0,0, \mathrm{P}), \mathrm{i}=1,2$, and the solutions corresponding to (8)-(9) are:

$$
\text { (30)-(31) } \quad \frac{1}{1+n} u_{1 C}^{\prime}\left(C_{1}, H, P\right)=\frac{n}{1+n} u_{2 C}^{\prime}\left(C_{2}, H, P\right)=u_{H}^{\prime}\left(C_{1}, H, P\right) \text {. }
$$

Again the $u_{H}$ are normalized to be identical at equilibrium. Simplify also by setting cross second derivatives with respect to $\mathrm{C}_{\mathrm{i}}$ and $\mathrm{H}, \mathrm{u}_{\mathrm{iCH}}$, equal to zero; other cases complicate without yielding further significant insights. The main new issue is how a
change in $P$ changes the bargaining solution and thus the equilibrium distribution of consumption between the two spouses. This in turn affects individual WTP for a changed supply of $P$. In principle such an effect could arise for two different reasons. First, relative inside bargaining positions of the two members could be affected. Secondly, outside options could be affected. Here we focus on the former effect. This implies an assumption (as in the main body of the paper above) that outside options are not exercised at equilibrium and that the inside utility level is always higher than the outside option level (e.g. from breaking up a marriage).

Consider now the effects of an increase in P on $\mathrm{C}_{1}, \mathrm{C}_{2}$ and H from changes in the bargaining solutions (30)-(31). Since such an increase in P leaves R constant, $\mathrm{dH}=-$ $\mathrm{dC}_{1}-\mathrm{dC}_{2}$, from (1). Differentiating (30)-(31) with respect to $\mathrm{C}_{1}, \mathrm{C}_{2}$ and H then yields the following approximate solutions: ${ }^{15}$

$$
\begin{equation*}
\frac{d C_{1}}{d P}=\frac{1}{D}\left[\left(u_{1 H P}-\frac{1}{1+n} u_{1 C P}\right)\left(\frac{n}{1+n} u_{2 C C}+u_{2 H H}\right)-\left(u_{2 H P}-\frac{n}{1+n} u_{2 C P}\right) u_{1 H H}\right] \tag{32}
\end{equation*}
$$

$$
\begin{equation*}
\frac{d C_{2}}{d P}=\frac{1}{D}\left[-\left(u_{1 H P}-\frac{1}{1+n} u_{1 C P}\right) u_{2 H H}+\left(u_{2 H P}-\frac{n}{1+n} u_{2 C P}\right)\left(\frac{1}{1+n} u_{1 C C}+u_{1 H H}\right)\right] \tag{33}
\end{equation*}
$$

$$
\begin{equation*}
\frac{d H}{d P}=\frac{1}{D}\left[\left(\frac{1}{1+n} u_{1 C P}-u_{1 H P}\right) \frac{n}{1+n} u_{2 C C}+\left(\frac{n}{1+n} u_{2 C P}-u_{2 H P}\right) \frac{1}{1+n} u_{1 C C}\right], \tag{34}
\end{equation*}
$$

where

[^10]\[

$$
\begin{equation*}
D=\left(\frac{1}{1+n} u_{1 C C}+u_{1 H H}\right)\left(\frac{n}{1+n} u_{2 C C}+u_{2 H H}\right)-u_{1 H H} u_{2 H H} \tag{35}
\end{equation*}
$$

\]

is a positive determinant. While this solution appears rather complicated in general, we may illustrate its main properties by considering two relevant special cases.

Case I: $u_{1 C P} \neq 0, u_{2 C P}=u_{i H P}=0, i=1,2$. Here the supply of the public good affects the marginal utility of private consumption for household member 1 but no other marginal utilities. In this case, (32)-(34) simplify to

$$
\begin{equation*}
\frac{d C_{1}}{d P}=\frac{1}{D}\left[-\frac{1}{1+n} u_{1 C P}\left(\frac{n}{1+n} u_{2 C C}+u_{2 H H}\right)\right] \tag{36}
\end{equation*}
$$

$$
\begin{equation*}
\frac{d C_{2}}{d P}=\frac{1}{D} \frac{1}{1+n} u_{1 C P} u_{2 H H} \tag{37}
\end{equation*}
$$

$$
\begin{equation*}
\frac{d H}{d P}=\frac{1}{D} \frac{n}{(1+n)^{2}} u_{1 C P} u_{2 C C} . \tag{38}
\end{equation*}
$$

Consider $u_{1 C P}>0$, whereby an increase in the supply of $P$ raises the marginal utility of the private good for member 1 only. This leads to higher private consumption for member 1, and to lower common household consumption and private consumption for person 2 . The utility change for member i when P is increased can be expressed as

$$
\begin{equation*}
\frac{d U_{i}}{d P}=u_{i C} \frac{d C_{i}}{d P}+u_{H} \frac{d H}{d P}+u_{i P}, i=1,2 \tag{39}
\end{equation*}
$$

In the case considered here, the sum of the two first terms is positive for member 1 and negative for member 2 . Consequently, member 1 is willing to pay more for the increase in P , than what appears from the analysis in section 2 above.

Intuitively, the increase in marginal utility of private consumption for member 1 makes it efficient for the household to also increase member 1's private consumption (in order to fulfil the efficiency conditions (30)-(31)). As a more concrete example, consider the case where spouse 1 only is a potentially eager golfer, and the increase in P is the building of a public golf course nearby, making golf a new option. This raises the marginal utility of private consumption for the golfing spouse, leading the household to allocate more of the common resources to this spouse's golfing hobby. (Thus in this particular case, spouse 1's increase in utility due to a higher P is greater than that of spouse 2 for two separate reasons: first, because the direct utility effect, $\mathrm{u}_{1 \mathrm{P}}$, is much higher, but also because the indirect effect via the household bargaining solution is positive.) Examples where the marginal utility of private consumption is lowered when P is increased are perhaps easier to find. Assume that spouse 1 has a medical problem that requires private treatment in the absence of a publicly available treatment, and assume that the increase in P implies that such a public treatment becomes available. Then spouse 1's marginal utility of private consumption is lowered, implying that the household is willing allocate less of its common resources to such costs for spouse 1 . This implies that the overall utility change, and WTP, for spouse 1 is smaller than that found in section 2 (in the concrete example, though, the total utility effect of the change in P may still be much higher for spouse 1 , which after all has the benefit of the treatment; the main overall positive effect for spouse 2 may be that private and household consumption are raised).

Case II: $u_{\underline{\underline{H P}}} \neq 0, u_{\underline{i C P}}=u_{\underline{2 H P}}=0, i=1,2$. Here an increase in P affects the marginal utility of H , for spouse 1 only. The effects on consumption are now

$$
\begin{equation*}
\frac{d C_{1}}{d P}=\frac{1}{D} u_{1 H P}\left(\frac{n}{1+n} u_{2 C C}+u_{2 H H}\right) \tag{40}
\end{equation*}
$$

$$
\begin{equation*}
\frac{d C_{2}}{d P}=-\frac{1}{D} u_{1 H P} u_{2 H H} \tag{41}
\end{equation*}
$$

$$
\begin{equation*}
\frac{d H}{d P}=-\frac{1}{D} \frac{n}{1+n} u_{1 H P} u_{2 C C} . \tag{42}
\end{equation*}
$$

Here a positive $u_{1 H P}$ reduces $C_{1}$, and increases $C_{2}$ and $H$. Overall utility of private and household consumption is now reduced for member 1, and increased for member 2. This is the opposite of what was found in case I. Here, when the marginal utility of common household consumption is increased for member 1, the marginal utility of private consumption should also be raised for that individual. This in turn implies that overall private consumption must fall for that individual. (Private consumption will be raised for the other individual, since H is raised and (30)-(31) must still hold.)

Consider two concrete examples. In the first, assume that only spouse 1 suffers from noise or air pollution, and that the increase in P takes the form of reductions in such pollution. This reduces spouse 1's utility from having an expensive house location (in an expensive neighborhood with low pollution levels to start with), or from taking strong defensive measures against pollution such as air filtering and window glazing. In this case $u_{1 H P}<0$. The Nash bargaining solution then dictates that

H be lowered (the couple moves to a less expensive location, or to a house with less defensive equipment). Private consumption for spouse 1 is increased (in order to bring the marginal utility of $\mathrm{C}_{1}$ down, in line with the reduction in the marginal utility of H for spouse 1). Then spouse 1's utility, and thus WTP for the increase in P, are raised by these consumption reallocations.

In the other example, assume that a husband (member 1) benefits from his wife's production of certain common household services, such as common meals or home cleaning, and that an increase in the public good reduces these benefits (while the effect is neutral for the wife; it could e.g. be the case that the husband now gets access to free meals or cleaning services with his empoyer, which makes him use there services instead of the previously-used common-household services). Then also here $u_{1 H P}<0$, while $u_{1 H P}=0$. By the same argument as above, the husband's effective relative bargaining power is increased when the public good supply increases. His equilibrium utility is raised beyond that found under separability, and his WTP for increases in the public good raised accordingly, while the wife's WTP is lowered.

## 6. Conclusions

Efficient household bargaining over private and common-household goods has important implications for how members of multi-person households tend to value public goods in contingent valuation (CV) questionnaire surveys. When one such member is asked to value a public good "on behalf of oneself", this valuation is likely to trade off the public good against that person's private-good consumption. When the person is instead asked to provide a public-good value "on behalf of the household", the public good is instead likely to be traded off against common-household goods. Under efficient Nash bargaining between two household members, and provided that
answers to CV questions are truthful, individual members always on average value the public good correctly "on behalf of the household", in the sense that individual expressed valuation equals the sum of individual household members' values "on behalf of themselves" on average. The member conducting the valuation overvalues (undervalues) the good on behalf of the household only when he or she has a higher (lower) marginal valuation of the public good than the other member. We also show that such over- or under-valuation is less serious under altruism, either pure altruism or greater mutual "paternalistic altruism" with respect to consumption of the public good.

These results have profound implications for the interpretation of results from CV surveys, which are today probably the most popular tool for valuing public goods. Most importantly, under my assumptions there is no basic problem with letting one person conduct such valuation on behalf of the household. In a large random sample, such valuations will, on average, correctly represent the respective households whenever they are truthful. Summing up individual valuations made on behalf of households across household members will then lead to double counting. Altruism plays no basic role in establishing these results. Still, higher degrees of altruism tend to make individual valuations within a household more equal, and one person's valuation on behalf of the household more precise.

Several extensions can be interesting for future analysis. Further research is required to determine the relevance of the unitary, bargaining and conflict views on household allocation. ${ }^{16}$ Secondly, many goods are on a more diffuse continuum, between our extremes of purely private or household goods. Thirdly, children and their preferences should be incorporated more directly, perhaps as individuals with

[^11]low bargaining power and sharing the common household budget, and subject to altruism (by parents). The analysis under either of the two altruism variants considered (or a combination) may then apply. Fourthly, altruism outside of the household may play important roles, not explored here.

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[^0]:    ${ }^{1}$ For a presentation of the CV method see Mitchell and Carson (1989). For an overview of recent applications and developments of the method, see Carson, Flores and Meade (2001).

[^1]:    ${ }^{2}$ See Bergstrom (1997) for a survey. See also Browning and Chiappori (1998) and Aura (2002) for recent empirical evidence in favor of the household bargaining model.
    ${ }^{3}$ In Quiggin (1998) one interpretation of the "public" good is as a good consumed exclusively within the household. He however does not clarify the implications of this assumption for the equilibrium allocation of the "public" good, which is the main issue here.
    ${ }^{4}$ In our model "public goods" also comprise private goods whose costs are not charged to users, as is the case for many educational, health and cultural services.
    ${ }^{5}$ We ignore the possibility of family break-up or outside options yielding higher utility than our bargaining solution. A divorce option is considered by Manser and Brown (1980) and McElroy and Horney (1981), and an "inside" noncooperative breakdown option by Lundberg and Pollak (1993). In our model the threat point is essentially immaterial as long as there is no breakdown of cooperation. Arguably, when household goods command a large share of the common budget, the divorce option may be unattractive even for spouses with low bargaining power, since this is likely to go together with low relative income when living as a single.

[^2]:    ${ }^{6}$ Asymmetric information about household members' individual income contributions may complicate this problem relative to our exposition. Arguably asymmetric information is a small problem in households where members interact daily. It may e.g. be difficult for one member to maintain a high consumption level without the other member discovering this.

[^3]:    ${ }^{7}$ This is not a restrictive assumption, at least not locally in the vicinity of the preferred solution. Since the utility functions utilized here fulfil standard vNM criteria of invariance to an increasing linear transformation, we may without loss of generality normalize to set marginal utilities of common household consumption equal at this point.

[^4]:    ${ }^{8}$ My model assumes that the "Coase theorem" holds for intrafamily allocations. In my view, if this

[^5]:    theorem is to hold approximately anywhere, this is likely to be in intrafamily contexts where the setting

[^6]:    is typically cooperative and individuals interact almost continuously.

[^7]:    ${ }^{9}$ We consider only intrafamily altruism. As justified e.g. by Becker (1991) and Jones-Lee (1992), this is likely to be the dominating type of altruism for a majority of individuals.

[^8]:    ${ }^{10}$ A third relevant alternative, important in practical CV applications but not pursued here, is to base the WTP question on an assumption that all individuals are to pay the same amount toward the public good.
    ${ }^{11}$ This conclusion is identical to Result 1 in Quiggin (1998); see also Johansson (1994) for similar results.

[^9]:    ${ }^{12}$ I here adopt Quiggin's (1998) terminology; perhaps a better term would be "public-good focussed" altruism as used by Jones-Lee (1992) and as also suggested by a referee.
    ${ }^{13}$ We are ignoring possible selfishly motivated altruism related to personal or household goods. For household goods such effects could be present if increased consumption of common household items develops preferences that are more similar among family members, or improves the other person's health status or longevity (as could be the case with a better dwelling, located in a safer and cleaner place, or better prepared common meals). For purely private goods, opposite paternalism is possible where one member prefers the other member to consume less of particular goods (as when the other member overeats, drinks or smokes, or spends time in some hazardous activity).
    ${ }^{14}$ (26) implies that the other member's utility of public-good consumption (and thus not the consumption level) enters the first member's utility function. This is a simplification which can be motivated if $z_{i}(P)$ represents member i's actual use of a public good such as cultural or health services.

[^10]:    ${ }^{15}$ These solutions are not exact since n , and thus the "effective bargaining parameters" $1 /(1+\mathrm{n})$ and $\mathrm{n} /(1+\mathrm{n})$, are also generally affected, while in these calculations I take n to be a constant. The qualitative effects will however be correct. The reason is that the adjustments of the $\mathrm{N}_{\mathrm{i}}$ and the respective effective bargaining parameters go in opposite directions. Thus when e.g. member 1's net utility $\mathrm{N}_{\mathrm{i}}$ goes up in the new solution, ceteris paribus, his or her relative bargaining strength is reduced somewhat to eliminate part of this increase.

[^11]:    ${ }^{16}$ A recent study based on bargaining but emphasizing a conflict view is Anderson and Baland (2002).

