Valuing statistical lives from observations of speed limits and driving behavior

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Abstract

The paper discusses how to derive empirical estimates of the value of a statistical life (VSL) from observations of highway driving speeds, and from how such speeds are affected by speed limits and penalties for speeding. When drivers optimize with respect to driving speeds, we discuss three alternative approaches. The first two rely on constructing drivers’ utility functions, and the last on revealed government preferences similar to that used by Ashenfelter and Greenstone (2002) (A-G). The two last approaches are based on observations of changed driving speeds when speed limits and speeding penalties change. When drivers are law obedient and adhere to speed limits only the A-G approach can be used. Their approach is however unrealistic in putting overly great demand on government information about VSL, and in addition provides upwardly biased average VSL estimates.
1. Introduction

Highway driving is risky in exposing drivers and their passengers to risk of accidents resulting in material damages, injury or death. Rational drivers will tend to select speeds that balance the gains from more rapid transportation, against the expected private losses due to greater accident risk when speeds are increased. These choices in principle embed a loss of value suffered (by the driver and possible passengers) in the event of a fatal accident.

The main purpose of this paper is to design a theoretical framework for identifying the value of such losses to drivers, from observations of driving speed patterns and how such patterns change when certain external factors, such as speed limits and fines for speeding, change. This may in turn be helpful in deriving values for a statistical life (VSL), as evaluated either by drivers themselves or by authorities with influence over speed limits and fine levels. As is widely recognized, reliable estimates of average VSL levels in the population are important for several reasons, such as for selecting correct levels of public investments in the transportation sector and elsewhere the economy. A number of approaches are today applied for assessing such values, hereunder both revealed preference (RP) and stated preference (SP) methods. The most used RP approach is to consider compensation required to accept more risky jobs. See the empirical studies by Garen (1988), Leigh (1995) and Shogren and Stamland (2002), and Mrozek and Taylor (2002) and Viscusi (1993) for overviews. Other related studies are based on road traffic behavior (such as propensity to wear seatbelts), and on behavior concerning various other types of protective measures (e.g. installing fire detectors); see de Blaey (2000) and Blomquist (2001).

In a recent paper, Ashenfelter and Greenstone (2002) (hereafter A-G) approach the VSL issue in a somewhat different way, using what we denote by “implicit
Local highway authorities are here assumed to know the true average values of VSL, valid for their particular communities, and adjust speed limits optimally in view of this value when allowed to do so by a more central authority. Using this approach A-G derive an upper bound on average VSL for (40) states in the U.S. that increased the speed limit from 55 to 65 mph on rural highways in 1987, when allowed to do so, and a lower bound on the (7) states that did not change this limit. Their model however has a number of weaknesses. For one thing, they ignore other accident costs than mortality risks, which tend to bias their VSL assessment in an upward direction. They are also unclear on whether or not changes in driving patterns in response to changed speed limits follow from optimal driving behavior. Their assumption that states know true VSL and act optimally in response to this, while controversial, is neither discussed nor substantiated in their presentation.

In sections 2-3 below we present a model, which expands on the A-G analysis in several directions. First, we here assume that drivers are not inherently law-abiding, but instead adjust their driving speeds optimally in response to changes in speed limits and to associated penalties for speeding. On this basis we construct, in section 4 of the paper, three alternative measures of VSL that can all, potentially, be implemented empirically, provided only that certain parameters related to drivers’ time costs, driving speeds and individual and aggregate accident cost functions can be observed. The first measure relies on structural identification of the utility function of the average rational driver, from observation of average driving speeds in response to average fines for speeding, and assumptions about the time costs of traveling. The second approach identifies the utility function from a different type of observation, mainly observed adjustment of driving speeds to changed speed limits and the

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1 “Implicit” here refers to values that are implicitly embedded in public-sector decisions when selecting public policy. See Carlsen, Strand and Wenstøp (1993) for an application to hydropower development.
associated penalties for speeding. The third measure is essentially that used by A-G and is based on observed adjustments in speed limits by local authorities, in situations where the authorities gain freedom to set such limits, and where we assume that the authorities know the true average VSL level.

In section 5 we present a different model where all drivers are instead assumed to be law-abiding and never exceed the set speed limit (while the unconstrained optimal speed would be higher than this limit). We then show that the two first approaches can no longer be used for identifying VSL, and the A-G approach is the only viable one. Our analysis here at the same time indicates that the interpretation of changes in average driving speeds under the second approach (where observations about such changes are used to recover the driver utility function directly) is sensitive to the distribution of drivers, between inherently law-abiding and optimizing types.

This paper opens up for some new approaches to the VSL issue, and identifies certain necessary key parameters in each case. We find that the A-G approach not necessarily requires less information on driver- and driving-related variables than the two other approaches that are appropriate (and which do not require rationality on behalf of government) when drivers optimize. We identify two main further weaknesses with the A-G approach. The first is their assumption that local governments know the true VSL levels, and systematically compare all costs and benefits accruing to drivers when speed limits change. This is not likely to hold in practice; indeed, had governments (local or other) systematically known the respective true VSL levels and systematically embedded this knowledge into their policy decisions, further research to uncover such levels would be unnecessary. Secondly, the A-G formulation implies biases (in particular, omitted accident costs in Norway.)
other than fatality costs, and upward bias of time costs) that in turn imply too high (individual-based) VSL levels.

A separate issue not dealt with here is whether true VSL values at all can be derived from individual-based studies, i.e. from studies where one infers individuals’ valuations of changes in mortality risk for oneself only, as argued in Strand (2003), interaction effects between individuals (hereunder altruism and various consumption interactions) imply that VSL is then typically underestimated, and that the values derived under the procedures suggested here must be viewed as (biased) lower bounds to VSL and not unbiased estimates. This in turn ties up with the long-standing dichotomy between VSL based on willingness-to-pay concepts, and the QALY concept largely favored by health economists; see Hammitt (2002) for a recent exposition of this debate.

2. The basic model with optimizing drivers

We start with considering an individual (or a set of identical individuals) who acts optimally (without moral restraint) in response to a given speed limit, and whose unconditionally optimal speed, \( S_e \) (chosen with no enforcement by the authorities), exceeds the existing speed limit \( L \). Denote the individual’s expected discounted utility as viewed from period \( t \) by \( EU(S(t),L(t)) \), to indicate that both the (endogenously) selected driving speed \( S \), and the (exogenous) speed limit \( L \) (with an associated, exogenously given, policy for its enforcement) influence on utility. The role of the authorities is viewed as that of implementing the speed maximizing \( EU(S(t),L(t)) \).²

² The model here assumes that individual trips involve only one occupant of each vehicle. According to A-G, the average number of persons occupying a car during a typical trip in the U.S. is 1.7. A generalization to more than one occupant is here straightforward given that neither individual accident and death probabilities nor individual valuations of time depend on the number of drivers.
Assume that individuals are risk neutral with constant death probabilities, and define the Bellman equation

\[ EU(S(t), L(t)) = C(t) + \delta \left[ 1 - p(S, S_{av}) \right] EU(S(t+1), L(t+1)), \]

where \( C(t) \) is consumption in period \( t \). The individual is assumed to die at the end of any given period with probability \( p \) affected by the possibility of suffering a fatal traffic accident. Ignoring other factors than those related to road accidents, we specify this probability as a function \( p(S(t), S_{av}(t)) \). \( S \) is the driving speed chosen by the individual, \( S_{av} \) the average driving speed chosen by all drivers, and \( \delta \) is a periodic discount factor. We assume that \( p_1(S, S_{av}), p_2(S, S_{av}), p_{11}(S, S_{av}), p_{22}(S, S_{av}), p_{12}(S, S_{av}) \) all are positive (foot scripts referring to first- and second-order derivatives with respect to the two arguments). The greater the speed of an individual, the higher the probability that the individual suffers a fatal accident, and this probability increases more for higher speeds. Likewise, when others drive at higher average speeds \( S_{av} \), the likelihood that one of them will crash with a given driver increases, for a given speed of this driver. In the following we assume that \( C(t) \) is constant over time, such that the time indicator may be dropped in (1). Define \( C \) by

\[ C = R + H + A - \alpha wT - F(S - L) - G(S, S_{av}), \]

\(^3\) Drivers are thus assumed to be risk-neutral with respect to income. This may be a realistic assumption when considering the small changes in wealth involved in realistic changes in road traffic mortality risk relevant here.

\(^4\) The probability of a traffic accident may comprise only a small fraction of the overall probability \( p \) of dying; the point here is only that this overall probability is affected by driving speeds.

\(^5\) A related externality issue, not pursued further here, is that when the other vehicle with which you may crash is heavier, the expected damage on yourself is greater; while the opposite may be the case when your own vehicle is heavier.
where \( R \) is a basic (exogenous) per-period income, \( H \) is a lump-sum net transfer to the individual from the government, and \( A \) can be viewed as the net utility value (apart from consumption) of being alive instead of being dead (where \( A \) is normalized to zero in the dead state). \( T \) is travel time and \( w \) is the individual’s wage rate. \( wT \) is thus labor income foregone while traveling, and \( \alpha \) is a factor converting the time cost of travel into net consumption loss (where consumption also includes leisure).\(^6\)

Typically, \( \alpha \) will be below unity, although we will open up for other possibilities.\(^7\) We have the following basic relationship between \( T \) and \( S \):

\[
T = \frac{D}{S},
\]

where \( D \) is total distance driven. Following A-G we assume that \( D \) is independent of the speed limit and of efforts to enforce it.\(^8\) \( F \) represents expected fines for speeding, assumed a continuous function such that \( F(0) = 0, F' > 0 \) and \( F'' > 0 \) for \( S>L \) (the individual is fined only when speeding, and the marginal expected fine increases by more for higher speeds). \( G \) is expected materials damage to oneself from non-fatal accidents, which depends on own and others’ speeds in the same qualitative (but not necessarily quantitative) way as for fatal accidents (i.e., \( G_1, G_2, G_{11}, G_{22} \) all positive).\(^9\)

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\(^6\) Travel time is the only direct cost component affected by driving speeds in the model. This appears to be a good empirical approximation, see e.g. Ghosh et.al. (1975).

\(^7\) A-G argue that \( \alpha \) should be unity of perhaps even higher, referring to a much-quoted study by Deacon and Sonstelie (1985). Most of the transportation literature however points to time costs well below wage costs, see e.g. Walters (1996) for further references.

\(^8\) Greenstone (2002) studies the relationship between average distances travelled and speed limits in states in the U.S., and finds no effect from an increase in the speed limit from 55 to 65 mph, in those states where changes were enacted.

\(^9\) We do not explicitly model accidents with personal injuries that do not result in death, but that may still reduce future life expectancy or the overall value of future life years (as implied e.g. by the QALY concept). \( G \) may however still be interpreted as incorporating such effects. Note that A-G do not consider costs of non-fatal accidents, probably on the presumption that such costs are inconsequential relative to fatal accidents. This is clearly true on a per-fatal-accident basis. The frequency of non-fatal accidents (considering both those with and those without personal injury) is however several orders of magnitude above that of fatal accidents.
In the following we assume for simplicity that individual drivers pay for all real costs associated with materials damages to themselves, but pay for no damages that may be inflicted on other vehicles (we also assume that none pays direct compensation to others for fatal accidents).\textsuperscript{10}

With time-invariant \( C \) and \( p \), (1) – (2) can be solved for \( EU(S, L) \) as follows:

\[
EU(S, L) = \frac{1}{1 - \delta \left[ 1 - p(S, S_{av}) \right]} \left[ R + H + A - \alpha w T - F(S - L) - G(S, S_{av}) \right].
\]

The individual driver is assumed to set his or her optimal speed to maximize (4) with respect to \( S \), leading to \( dEU(S,L)/dS = 0 \) and the following first-order condition:

\[
\delta p_{1}(S, S_{av}) EU(S, L) + G_{1}(S, S_{av}) + F'(S - L) = \alpha w \frac{D}{S^2}.
\]

Equilibrium with identical drivers implies \( S_{av} = S \) in (5). We note that \( \alpha w D / S^2 = TC/S \), where \( TC \) is the total time cost of traveling. Condition (5) states that the sum of incremental costs for the individual, associated with increasing driving speed, on the left-hand side, must equal the incremental time cost saving from driving faster, on the right-hand side. The incremental private cost has three components. The first is the effect on discounted lifetime utility from an increase in the probability of a fatal accident; the second is an increase in expected costs associated with nonfatal accidents; and the third is an increase in expected fines for speeding.

\textsuperscript{10} Typical examples where individuals do not pay for their full accident cost would be where their car is insured against such accidents and insurance premia are not sufficiently experience rated; or when one...
We concentrate on the case $S > L$, which always holds whenever $S_e > L$, as was assumed above, which in turn implies $F' > 0$ (the equilibrium speed will exceed the speed limit and the individual fined for speeding when controlled). In an opposite case of $S \leq L$, the speed limit would not affect the behavior of drivers. In this case $S_e = S$, $F \equiv 0$ (no fines for speeding are relevant) and the government has no instruments for affecting speeds. We will later see that, in general, the government always wishes to set $L < S$ in order to implement efficient driving speeds (regardless of the unconstrained optimal speed $S_e$), given its instruments represented by the $F$ function, something that justifies our assumption that the speed limit is below $S_e$.

We can now study how an increase in $L$ affects equilibrium driving speeds. We then differentiate (5) with respect to $S$ and $L$ (setting $S_{av} = S$), recognizing that $dEU(S, L)/dS = 0$ by virtue of the first-order condition (5). This yields:

\[
(6) \quad \frac{dS}{dL} = \frac{F'' \frac{\delta p_1}{1 - \delta(1 - p)} F'}{F'' + \Gamma},
\]

where

\[
(7) \quad \Gamma = 2 \frac{\alpha w D}{S^3} + G_{11} + G_{12} + \delta(p_{11} + p_{12})EU(S, L).
\]

Note that $p_1$, the increase in fatal accident probability when driving speed increases marginally, is likely to be very small relative to the other parameters included in (6).
We can then ignore the second term in the numerator of (6), and write (6) on the form\(^{11}\)

\[
\frac{dS}{dL} = \frac{F''}{F'' + \Gamma} \equiv \Phi F''.
\]

Since \(\Gamma > 0\), \(dS/dL\) from (6a) is positive but less than unity, implying that an increase in the speed limit leads drivers to drive faster on the average, but the increase is smaller than the increase in the speed limit. Given that driving speeds are chosen optimally, and that the unconstrained optimal speed (in the case of no fines) is higher than \(L+\Delta\) (the speed limit after the change), we then always have \(\Phi \in (0,1)\).

### 3. Socially optimal speed and speed limits

We may now derive the socially optimal speed \(S\), with an associated optimal speed limit \(L\), assuming that the government is constrained to use the enforcement function \(F\). Note then first that if the traffic authority would have at its disposal a function \(F\) which was increasing at (close to) an infinite rate in the neighborhood of \(L\), we see from (6) that \(dS/dL = 1\) in this case. Then the authority would be able to directly impose the speed limit \(L\) on all drivers (nobody would ever choose to go above the limit, as this would be too costly in terms of additional fines or other types of punishment).\(^{12}\) This is the same as when drivers are law obedient, as will be considered in section 5 below.

\(^{11}\) One might here immediately think that \(F''\) could have a similarly small value thus making our argument here invalid. It can however easily be shown that \(F''\) must be positive at an internal optimal solution for drivers in this model, and not just marginally so under the case where driving speeds are affected in a significant way by speed limits.

\(^{12}\) This would also require that the maximum fine be sufficiently high to deter speeding. Note that we are here not analysing the issue of the optimal combination of fines and detection probabilities. According to Becker’s (1968) seminal analysis, when enforcement is costly and individuals are risk
Consider the government’s problem where the utility of a representative individual is maximized with respect to $S$. The objective function for this problem can be defined in the same way as (4), except that $H \equiv F(L-S)$ (the representative individual is always paid back the traffic fines imposed), and where we now assume that $S_{av} = S$. Maximizing $EU(L)$ under such assumptions yields the first-order condition:

\[
\delta[p_1(S, S) + p_2(S, S)]EU(L) + G_1(S, S) + G_2(S, S) = \alpha \omega \frac{D}{S^2}.
\]

The optimal solution is implemented if and only if

\[
\delta p_2(S, S)EU(L) + G_2(S, S) = F'(S - L).
\]

The slope of the penalty function $F$ should here be set to equal the sum of the marginal external cost components, involving the impact on other drivers. There are two such cost components, the first associated with loss of life in fatal accidents, and the second with other accident damage (involving both fatal and non-fatal accidents).

4. Deriving values for statistical life

We will now attempt to use the above model to derive values for statistical life (VSL), as valued by the individual in question. This is a somewhat limited scope for VSL valuation, as it abstracts from all types of inter-personal effects in such valuation that may arguably be important; see Strand (2003) for a presentation of arguments. Still, neutral, the optimal scheme implies that the probabilities of detection (and consequently the enforcement cost) can be reduced arbitrarily by only increasing fines sufficiently. This is clearly a more complex issue when individuals are risk averse, and more generally, when the legal system is designed not solely on the basis of pure efficiency principles.
the individual-base approach dominates the current literature (see Johansson (2001, 2003); Cameron and DeShazo (2003); Hammitt (2002)), and we follow this tradition here. From Thaler and Rosen (1976), Rosen (1989) and Viscusi (1993), VSL can be defined as the marginal rate of substitution between money and death risk. Using (4), we derive our measure for VSL, denoted V, as follows:

\[ V = \frac{dC(t)}{dp(t)} = \delta EU(t+1). \]

The \( \delta \) parameter relevant in expression (10) is here related to the time span from action (the act of driving) to effect (a possible fatal accident); in practice this time vanishes as fatal accidents are affected continuously by current driving. This implies that \( \delta = 1 \) is the relevant parameter in (10). Since \( EU(t+1) = EU(t) = EU \), we simply have \( V = EU \). This is clearly reasonable; \( EU \) is the overall present discounted value of staying alive over being dead for the individual, i.e. the person’s “value of life”.

In the following we will discuss three different ways of deriving estimates of VSL in our model, given that the relevant data are available. The first two imply that one seeks an estimate of \( EU \), to be inserted into (10). In the first case we use the relationship (5), where we must assume that all parameters except \( EU \) are observed. In the second case we require observations of changes in driving speeds in response to a change the speed limit. We then base the identification of \( EU \) on the relationships (6a) and (7), assuming that the parameter \( \Phi \) (the relative driving speed response) and all parameters entering into (7) are observed. The third procedure also requires that changes in driving speeds in response to changed speed limits are observed. We here however assume that the authorities at the outset know the true value of VSL, and that
they use this value to adjust the speed limit. An increase (decrease) in the speed limit from an initially lower level will then reveal whether VSL is lower (higher) than a particular level.

4.1. Case a: Deriving VSL directly from the first-order condition

In the first of the three cases the expression derived for EU, denoted EU(1), can be found from (5) simply as

\[
EU(1) = \frac{\alpha wD}{S^2} \left[ G_1 - F' \right].
\]

The corresponding VSL expression can be found, using (10), as

\[
V(1) = \frac{\alpha wD}{S^2} \left[ G_1 - F' \right].
\]

This expression can alternatively be written as follows:

\[
V(1) = \frac{1}{p El_1(p)} \left[ Tc - \theta_1 G - \varphi F' \right],
\]

where we have defined \( El_1(p) = p_1 S/p \), \( \theta_1 = G_1 S/G \) and \( \varphi = F' S/F \) as respective elasticities of the p, G and F functions. These elasticities are in principle observable together with p, G and F, given relevant data on accident frequencies. The cost parameter \( \alpha \) may not be immediately observable, but can be assessed through surveys.
To illustrate a possible calculation of VSL on the basis of (12a), consider a driver who spends half an hour daily behind the wheel (180 hours a year), earns an hourly wage of 16 USD, and that his or her $\alpha$ value equals $\frac{1}{2}$. Then TC = 1440 USD per year. Assume that $p$ equals the U.S. average of approximately 1/6000 annually. Assume also $E_l(p) = 2$, $\theta_1 = 2$, $\phi = 3$, while $F = 100$ USD. If now $G = 0$, our calculated VSL value equals $(6000/2)(1440-300)$ USD = 3.4 million USD. Assume instead that private accident costs associated with non-fatalities in expectation amount to 300 USD per year. We see that this lowers the calculated VSL value substantially (the last parenthesis in the expression is now lowered by 600), to 1.6 million USD. Assume instead (as A-G do) that $\alpha = 1$. Then in the first case, VSL = 7.7 million USD, and in the second case, 5.9 million USD. We note that these illustrative calculations appear to give at least the correct order of magnitude of VSL for the U.S. (they are close to the standard estimate of 5 million USD for the U.S., applied e.g. by Murphy and Topel (2003)).

4.2 Case b: Deriving VSL from observations of changes in driving speeds

Under this case the authorities are not assumed to necessarily choose a socially optimal level of $S$, but may instead view $S$ as a policy parameter to be changed, possibly for reasons outside of the model. Such changes can be implemented in two ways: by changing the speed limit $L$ for given fine structure; or by changing the fine function $F$ keeping the speed limit fixed.\(^{13}\) The most straightforward procedure in the context of our model is to change $L$ only. Assume that $L$ shifts upward by a small amount $\Delta$, to a new level $L_0+\Delta$ (from the initial value $L_0$). Then $S$ shifts up by an

\(^{13}\)Note that nothing will happen with driving speeds when only the speed limit is changed, given that drivers are not inherently law-abiding as assumed here. The penalty function for speeding has to
amount $\Phi \Delta$, where $\Phi$ is a number between zero and unity, given by the expression on the right-hand side of (6a). With knowledge of all parameters in (7), EU can in this case (denoted EU(2)) be recovered from the observation of $\Phi$, as follows:

$$EU(2) = \frac{1 - \Phi}{\Phi} \frac{F_n - 2awD}{S^3} \frac{G_{11} - G_{12}}{\delta(p_{11} + p_{12})}.$$  \hspace{1cm} (13)$$

The corresponding VSL value, $V(2)$, can now be found from (10) as

$$V(2) = \frac{1}{1 - \delta(1 - p)} \frac{1 - \Phi}{\Phi} \frac{F_n - 2awD}{S^3} \frac{G_{11} - G_{12}}{(p_{11} + p_{12})}.$$  \hspace{1cm} (14)$$

The following alternative form of (14) lends itself more easily to empirical implementation:

$$V(2) = \frac{1}{1 - \delta(1 - p)} \frac{1}{El(p_1)El(p_2)p} \left[ \frac{1 - \Phi}{\Phi} \phi \phi F - 2TC - \theta_1 \theta_1 G \right].$$  \hspace{1cm} (14a)$$

We have here introduced the notation $\phi_1 = F''S/F'$, $\theta_1 = (G_{11} + G_{12})S/G_1$ and $El(p_1) = (p_{11} + p_{12})S/p_1$. We have above shown that $\Phi \in (0,1)$, i.e., the driving speed is adjusted only partially in response to an increase in the speed limit.

In principle, $V(1)$ and $V(2)$ ought to yield the same VSL values for the same population. Being able to derive both these values can then be a useful consistency change such that a higher marginal penalty is imposed for a given speed. This is clearly the case with the function $F(S-L)$ assumed above, as this function shifts for a given speed $S$ when $L$ shifts.
check on the model, in particular since the parameters entering into the two expressions are quite different. Finding empirical estimates of \(V(2)\) is however more demanding than for \(V(1)\), in two separate ways. First, the number and types of parameters one needs to know is greater in this case. Secondly, one needs an “experiment” whereby drivers’ responses to changes in the speed limit are observed. We will thus not attempt to derive any concrete estimate for \(V(2)\) here.

4.3 Case c: Deriving VSL from government speed limit choice

We will now consider the approach to the VSL issue taken by A-G, namely to assume that authorities at the outset know the true value of VSL, and that they use this information when allowed to adjust the speed limit. When the speed limit \(L(0)\) is initially imposed on authorities (as in the case of the 55 mph limit in the U.S. prior to 1987), with associated average driving speed \(S(0)\), the authorities wish to implement an increase in the speed limit to a higher level when allowed to do so, say to \(L(1)\), with associated higher driving speeds \(S(1)\), given a positive overall social value of this change as viewed by the authorities. Differentiating (4) with respect to \(L\), and assuming budget balance (such that households are paid back their speeding fines in the form of lump-sum payments, \(dH = F'(dS-dL)\)), we find that the authorities are indifferent with respect to changing or retaining \(L\) given that

\[
\frac{dEU(S,L)}{dL} = \frac{1}{1-\delta(1-p)} \left[ \frac{\alpha wD}{S^2} - G_1 - G_2 - (p_1 + p_2)V(3) \right] F^n F^{n+\Gamma} = 0,
\]

where \(V(3)\) is the VSL value in this case. There is a positive inequality in (15) whenever it is strictly optimal for the authorities to raise the speed limit, and a negative inequality when it is strictly optimal to lower it. As noted VSL is here
assumed to be known, by the authorities and thus by the public. Equality of (15) requires that \( V(3) \) fulfill the following condition:

\[
V(3) = \frac{TC}{S} \left( G_1 - G_2 \right) \frac{p_1 + p_2}{p_1 + p_2}.
\]

Alternatively,

\[
(16a) \quad V(3) = \frac{TC - \theta G}{p El(p)}.
\]

We have defined \( \theta = (G_1 + G_2)S/G \), \( El(p) = (p_1 + p_2)S/P \). We see that (16a) tends to yield a lower value for VSL than (12a) when there are accident externalities, i.e. \( p_2 \) and \( \theta_2 \) are positive.

VSL is less than \( V(3) \) from (16) when there is a positive inequality in (15), and higher than \( V(3) \) when there is a negative inequality (since a low value for VSL makes it attractive for the government to increase the speed limit and vice versa).\(^{14}\)

We may compare the expressions \( V(3) \) in the present case to \( V(1) \) for the case where the calculation is made directly from observations of chosen vehicle speeds without necessarily assuming that the speed limit is set optimally. Manipulating the expressions lead to the following relationship:

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\(^{14}\) After the 1987 legislative change in the U.S., when the 55 mph speed limit no longer was enforced as a uniform national standard but states instead were free to increase their limit up to 65 mph on rural interstate highways, 40 states chose to increase this limit, while the rest did not; all the latter states were in the northeast corner of the U.S.. Ashenfelter and Greenstone argue that the “true” VSL value in these 40 states was thereby revealed to be lower than a value fulfilling an equivalent of our expression.
(17) \[ V(3) = \frac{p_1 + \beta p_2}{p_1 + p_2} V(1), \]

where \( \beta \) is defined by the relationship \( \beta p_2 V(1) = F' - G_2 \). \( \beta \) is a parameter indicating whether or not the speed limit is set optimally, and the degree of possible non-optimality. From (9), when \( \beta = 1 \) the speed limit is optimally chosen. We then find, as is intuitively reasonable, that \( V(3) = V(1) \), and that the VSL measures as derived from revealed individual behavior, and from revealed government preference, are identical. We also see that \( V(3) > (\leq) V(1) \) according to whether \( \beta > (\leq) 1 \). When \( \beta > 1 \), the speed limit is set too low (since \( V(1) \) then is low relative to marginal social costs of increased speed), and too high in the opposite case.

Clearly, when the government knows the true VSL level and can set the speed freely, \( S \) will be chosen optimally and we will have \( V(3) = V(1) \). This still of course begs the question of how the government can come to know the true VSL level.

We will also note some other sources of error in the individual-based VSL estimates derived by A-G. First, A-G ignore the \( G_i \) terms (representing non-fatal marginal accident costs), and secondly, travel costs are counted at the full working wage rate (\( \alpha = 1 \)). Both factors tend to increase their VSL estimate, and thus lead to a higher estimate than that found from our expression (16). To get a feel for the error involved in the first of these assumptions, note that Miller (1993) has estimated that among total motor vehicle accidents cost the United States in 1988 (assessed at $333 billion), fatalities account for a mere 34%, while non-fatal injuries account for 53% (brain injuries being the largest component), and material damage and time delays for 13%. What motivates drivers in terms of driving speeds is here the internalised part of

(12) under their formulation, while the “true” VSL value was above this level for those states that chose to retain the speed limit.
these costs, i.e. those cost components for the individuals themselves that are affected by changes in individual driving speeds. There is here reason to believe that this share is not substantially higher for individual mortality risk changes than for material damage or damage from personal injury. Thus the A-G assumption on this point may lead to overestimating VSL with a factor or about 3. This in case implies that their estimates of maximum VSL for those states in their sample, that chose to increase the speed limit from the initial level, is too high. (There is on the other hand no automatic bias in the estimate of VSL for those states that do not change their speed limits. These have higher VSL than the derived level, but it is unclear how much higher.)

5. The model when drivers are law-abiding

The model and calculations above were based on an assumption that all drivers in the economy act optimally in response to changes in speed limits, and have no respect for these limits as such. Some may consider this to be unrealistic, in particular since certain drivers may be “inherently law-abiding” in the sense of adhering to the speed limit even when this is not necessarily in their best economic interest. We will in this section consider a different, equally extreme, alternative where all drivers are identical but instead law-abiding, in the sense that they never drive at a speed above the speed limit. For speed limits to be relevant, the unconstrained optimal driving speed of drivers must also here be higher than the actual speed limit L. Equations (1)–(4) are the same as before, except that now $S = L$ for all drivers in this category. When all drivers are law-abiding, $S_{av} = S = L$. Since there is no speeding, $F$ drops out, and $H = 0$ (as $H$ in the previous version represented refunding of paid speeding fines). The optimal speed limit to be set by the government is here simply the solution to (8), inserted $S = L$. 
In this case there are fewer possibilities for identifying VSL from data of the types discussed in section 4 above. In particular, changes in the behavior of drivers now cannot provide us with a value for VSL, since this behavior follows directly from the speed limit. We are left with alternative c, where the authorities know the VSL value. With identical drivers the relevant expression for EU is then found from (4), inserting $S = L$ (and dropping the $F$ function), as

\[
EU(L, L) = \frac{1}{1 - \delta \left[1 - p(L, L)\right]} \left[ R + H + A - \alpha w \frac{D}{L} - G(L, L) \right].
\]

Finding the optimal $L$ for the government now implies solving

\[
\frac{dEU(L, L)}{dL} = \frac{1}{1 - \delta (1 - p)} \left[ \frac{\alpha w D}{L^2} - G_1 - G_2 - (p_1 + p_2)V(3) \right] = 0,
\]

which is essentially identical to (15) for the case of optimizing drivers. Also here, the speed limit should be increased when $V(3)$ is below the level yielding equality in (19), and reduced when $V(3)$ is higher than this level.

Note that this approach does not require that individuals themselves know (or are aware of) their own true VSL, but that authorities know these values. The point here is that it is not the individuals’ awareness of their exact VSL that makes them change their driving speeds; these speeds are invoked directly by the changed speed limits. Thus whatever (more precise) information individuals have about their own VSL has no direct influence on their behavior in this case, making it impossible to recover VSL estimates from changes in driving behavior.
A relevant issue is how the various relationships should be interpreted in the more realistic case where positive fraction of drivers are inherently law-abiding, while the rest are optimizers. Note then that when the parameter $\Phi$ is observed on average for law-abiding and non-law-abiding drivers, we find that $\Phi = \beta \Phi_1 + 1 - \beta$, where $\Phi_1$ is the (average) value of this parameter for optimizing drivers, and $\beta$ and $1 - \beta$ are the fractions of optimizing and law-obedient drivers respectively. Thus $\Phi_1 < \Phi$, and the observation of the average $\Phi$ will yield biased estimates of VSL, using (14) (in case b above).

6. Conclusions and final comments

We have above studied a model of driving behavior and responses of such behavior to speed limits and efforts to enforce these limits. We show that when drivers select their driving speeds optimally, balancing expected private time costs against expected damage costs, speeding fines and probabilities of sustaining fatal accidents, estimates of the average value of statistical life (VSL) among highway drivers can in principle be recovered in three different, alternative ways. The first, and simplest, relies on data for observed driving patterns, together with the necessary data on (internalized) costs of accidents. The second, more demanding, procedure relies on observations of changes in driving patterns when speed limits (together with associated fines for speeding) change. Both these depart from assumptions that individuals themselves, but not the authorities, know the true VSL. The third approach, recently applied in an empirical study by Ashenfelter and Greenstone (2002) (A-G), by contrast assumes that the authorities (but not necessarily individuals themselves) know true VSL, and on this basis derives an upper or lower bound for average VSL from observations of changed behavior among the authorities.
themselves; i.e., whether or not they choose to increase the speed limit whenever allowed to do so. In our opinion this is a far more doubtful approach than the former two: there appears to be no good reason why the authorities ought to know true VSL while individuals do not. We have also identified sources of bias in the A-G approach which can easily be corrected provided that the appropriate data are available.

The current note is sketchy and suggestive, and does not go far in the actual quantification of VSL. Our main point is to indicate that attempting at such quantification from data on highway driving patterns and changes in such patterns may be a fruitful undertaking, but then largely using different procedures than those suggested by A-G. We intend to take part in this research.
References


