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Optimal
Disease Eradication

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Abstract

Using a dynamic model of the control of an infectious disease, we derive the conditions under which eradication will be optimal. When eradication is feasible, the optimal program requires either a low vaccination rate or eradication. A high vaccination rate is never optimal. Under special conditions, the results are especially stark: the optimal policy is either not to vaccinate at all or to eradicate. Our analysis yields a cost-benefit rule for eradication, which we apply to the current initiative to eradicate polio.

1.   Introduction

The *eradication* of an infectious disease is an extreme—indeed, a singularly ambitious—policy goal. It is to be contrasted with a policy of *control*, which reduces incidence below the competitive level but not to zero, and a policy of *elimination*, which cannot stop disease imports but which can prevent a local epidemic. It is a goal that has been tried before (hookworm, yellow fever, yaws, malaria), but achieved only once (smallpox). It is a goal that is being attempted again now (poliomyelitis, dracunculiasis), and for which there exists a long wish list of future candidates (among them, mumps, rubella, lymphatic filariasis, cysticercosis, and measles). It has even been suggested that the newest infectious disease, SARS, be eradicated.

Why eradicate? Suppose that a disease can be controlled—say, by means of vaccination. Suppose as well that the disease is already being controlled, and at a very high level—so high, in fact, that a slight increase in the vaccination rate would cause the disease to be eradicated. Eradication would increase costs in the short run, and prevent a few additional infections. But in making the pathogen disappear, eradication would also avoid the need ever to vaccinate in the future: a huge "dividend." A very high level of control will therefore never be optimal. Intuitively, the optimal policy will require no control, a modest level of control, or eradication. In this paper, we develop this intuition formally.

A disease can only be eradicated if it is eliminated everywhere in nature. Hence, our analysis applies to two kinds of situation: at the global level and at the level of the nation state after every other country has already eliminated the disease. If countries were symmetric, it might seem that the calculus of eradication would be the same for both of these situations. Barrett (2003), however, shows that, depending on the costs and benefits facing the "last" country, global disease eradication—a global public good—may be either a coordination (weakest link) game or a prisoners' dilemma.[1] Though Barrett (2003) exposes the underlying incentive problem, his analysis relies

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[1] Of course, eradication could also be globally inefficient or it could be in every country's interests to eliminate the disease unilaterally. Neither of these possible cases is economically interesting. In a related paper, Cooper (1989) examines international cooperation in the control of cholera and the eradication of smallpox, arguing that successful cooperation hinges on whether knowledge of cause and effect exists. Our analysis, and the literature summarized in this section, presumes such knowledge.

on a static framework in which eradication is assumed to be instantaneous—an outcome that may not be optimal (or even feasible). Our paper focuses on the dynamics of eradication, solving explicitly for the conditions under which eradication (whether at the level of the globe or the "last" country) will be optimal.[2]

Geoffard and Philipson (1997) also develop a dynamic model of the economics of disease eradication, but their focus is the positive analysis of public vaccination policy being (partially) crowded out by market behaviour (under the assumption that the private demand for vaccination increases with prevalence). They also do not solve explicitly for the conditions under which eradication is socially optimal, let alone the optimal path to eradication. Indeed, in their model, eradication can only be achieved in the limit as time goes to infinity (their analysis only compares steady states).

In a model in which people are either susceptible or infected (never immune), and in which the control is treatment rather than vaccination, Goldman and Lightwood (2002) derive the conditions under which asymptotic eradication is an optimal steady state. Moreover, they show that the initial infection rate determines whether asymptotic eradication is optimal—a result also demonstrated here (see Section 4.3). However, there is no dividend to eradication in the Goldman-Lightwood framework, the focus of our inquiry. Such a dividend can only be realized if eradication is achieved in finite time. As noted by Gersovitz (2003), "An important question would be whether settling for an internal steady state with positive infection is dominated by a push for eradication in finite time." We address this question directly.[3]

The dividend from eradication can be enormous. According to Fenner *et al*. (1988), the annual global benefit of smallpox eradication was about $1.35 billion (using 1967 as a base year), while the total cost of eliminating smallpox from the remaining endemic countries was about $300 million. Assuming a three percent discount rate, the benefit-cost ratio for smallpox eradication was thus about 150:1. Taking into account only the incremental costs needed to eliminate smallpox from the remaining

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[2] Indeed, we shall show that, for the case of linear costs—the case actually studied by Barrett (2003)—it will be optimal to eradicate instantaneously, if allowed by the feasibility constraints.
[3] Related papers on the economics of vaccination, but not eradication, include Brito, Sheshinski, and Intriligator (1991), Geoffared and Philipson (1996), Francis (1997), Gersovitz (2003), and Gersovitz and Hammer (2003).

endemic countries ($100 million), the benefit-cost ratio was even higher: about 450:1. Smallpox eradication was thus an astonishingly good deal for the world. It was also a good deal for individual countries. The United States, for example, saved about $150 million annually because of smallpox eradication (Fenner *et al.,* 1988), mainly in the form of avoided vaccination costs. Again, using a three percent discount rate, the eradication dividend to the United States alone was about $5 billion, a small fraction of the (essentially, one-time) cost of eradication.

Smallpox was the ideal candidate for eradication: there were no long-term carriers; smallpox survivors were immune for life; infected persons were easily detected; and only persons showing symptoms (probably) could transmit the disease. Moreover, the disease was only mildly infectious (relative to some other diseases, that is; the disease could be eliminated by mass vaccinating "only" 80 percent of a population), and the vaccine was relatively inexpensive (a single injection offered effective immunization). Being a live vaccine, immunization was risky, so that the rich countries had a strong incentive to eradicate; and because smallpox killed around a third of infected individuals, poor countries also gained substantially from eradication.

Unfortunately, the eradication of other diseases is likely to be more difficult, and less attractive in benefit-cost terms. For example, though measles kills about three-quarters of a million children every year in developing countries, in rich countries, where the disease has been eliminated, the measles vaccine is given as part of a combined vaccine (measles-mumps-rubella or MMR) and the savings from eliminating just the measles component may be relatively small.[4] As well, measles is more infectious than smallpox, and eradication would require vaccination (in multiple doses) of a very high proportion of the population (probably 95 percent or greater)—a problem if marginal costs increased in the vaccination rate. As explained in Section 7, the epidemiology of polio eradication is also problematic, and the economics far from

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[4] Estimates of the savings from measles eradication vary. According to Miller *et al*. (1998), the net benefits of measles eradication to the US would be between $500 million and $4 billion (1997 dollars). Savings estimates by Carabin and Edmunds (2003) are in the range of $10 million and $623 million for a selection of rich countries (Canada, Denmark, Finland, the Netherlands, Spain, Sweden, and the United Kingdom). One reason for the lower savings estimated by Carabin and Edmunds (2003) is the assumption that vaccination at some level would need to continue even after the wild virus had been eradicated because of the threat of bioterrorism—an issue discussed in the next paragraph.

overwhelming, even though the current campaign has already eliminated the disease from most parts of the world.

The threat of bioterrorism further weakens the economic case for eradication. Countries may now feel the need to continue to vaccinate, even if at a relatively low level (Carabin and Edmunds, 2003, for example, assume that vaccination for measles would be reduced but not stopped even after eradication), or to stockpile vaccine, and prepare for emergency distribution in the event of an attack (the approach being by a number of countries, including the United States, with respect to smallpox). These kinds of measures shrink the eradication dividend, while probably having no effect on the economics of control.[5]

In summary, the economic calculus for eradication of the most favourable remaining candidate diseases is likely to be more finely balanced than for smallpox—meaning that our framework for benefit-cost analysis needs to be more carefully specified. Our paper is a contribution to this effort. We derive a cost-benefit rule for optimal eradication, and demonstrate its utility by applying it to the current global initiative to eradicate polio.

Our paper progresses as follows. Section 2 develops the epidemiological model that describes how eradication might be achieved in finite time, and Section 3 specifies our economic model of eradication. Section 4 solves for the optimal eradication policy, and Sections 5 and 6 analyse special cases. Section 7 applies our framework to the current effort to eradicate polio. Section 8 summarises our main results.

2.   Epidemiology

We take as our starting point Anderson and May's (1991) model of the dynamics of immunization, a standard in the epidemiology literature:[6]

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[5] The rich countries would presumably be the target of a bioterrorist attack, but the rich countries are likely to eliminate candidate diseases for eradication unilaterally, making further measures to defend against a bioterrorist attack unnecessary.

[6] There is one minor difference between our specification and Anderson and May's. Anderson and May (1991) assume that only newborns are vaccinated. We assume that any and all susceptible persons may be vaccinated, not just newborns.

(1) $\dot{x}(t) = m - [m + \lambda(t)]x(t) - p(t)$,

(2) $\dot{\lambda}(t) = (v + m)\lambda(t)(R_0 x(t) - 1)$,

where $x(t)$ is the fraction of the population that is susceptible, $\lambda(t)$ is the force of infection (the rate at which susceptible individuals become infected), and $p(t)$ represents the overall rate of vaccination (only susceptible individuals are vaccinated). This dynamical system assumes that population is constant (births equal deaths; we normalize by setting population equal to one), with m representing both the birth and mortality rate. It also assumes that the disease is non-lethal. The parameter v represents the rate at which infected individuals become immune. Finally, R_0 is the basic reproductive rate of the microparasite (for a disease to spread, it is essential that $R_0 > 1$). A brief explanation is given in Appendix A.

For our purposes, this system of differential equations poses a problem: If the system (1)-(2) begins at $\lambda(t) > 0$, it can only converge to $\lambda(t) = 0$ as $t \to \infty$. This wouldn't matter if we only needed to study steady states. However, as noted in the introduction, the reason for pursuing a policy of eradication, rather than of high control, is to reap the benefits of not having to vaccinate post-eradication. If the aim is to study the optimality of eradication, the dynamics must permit eradication in finite time.[7]

Just how to model this is not so obvious. To Gersovitz (2003), moving "away from the no-eradication property of the model would require a more cumbersome model of finite lives." Our approach is much simpler. We solve for the steady state, $\lambda^\infty = m(R_0 - 1) - pR_0$, and assume that the dynamics can be represented by an adjustment equation,

(3) $\dot{\lambda}(t) = \sigma[m(R_0 - 1) - p(t)R_0 - \lambda(t)]$

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[7] We need hardly add that smallpox *was* eradicated, and in a period of just 10 years. Empirically, (1)-(2) is invalid, at least for small λ.

for $\lambda(t) > 0$, where σ is the speed of adjustment parameter.[8] Conveniently, (4) captures (almost) everything we need in a single equation. For our purposes, $x(t)$ is not of direct importance. $x(t)$ is only important insofar as it affects $\lambda(t)$, and this effect is reflected in (3).

Of course, simplicity always comes at a cost. Eqs. (1)-(2) imply that a small increase in the vaccination rate will reduce the force of infection by more when λ is high than when λ is low (for a given value of x). In epidemiological terms, the number of follow-on infections prevented by a single vaccination increases with the force of infection. Our use of Eq. (3) fixes this effect. In economic terms, eq. (3) makes the marginal benefit of vaccination independent of the vaccination level. When comparing a policy of high control versus eradication, use of (3) will not distort matters very much. The simplification matters more when comparing a policy of low control versus eradication. This is especially important for our linear model, presented in Section 5, and we discuss this assumption again in this section.

Before presenting our economic model, one further adjustment is required. Our main interest lies not in $\lambda(t)$ but in the proportion of the population that is infected under a control program. Denote this proportion $y(t)$. Assuming homogenous mixing, $\lambda(t)$ will be proportional to $y(t)$ (Anderson and May, 1991). In particular, we can write $\lambda(t) = \beta y(t)$ where β is a transmission parameter. We can thus rewrite (3) as

$$(4) \qquad \dot{y}(t) = \sigma[\tilde{R}_0(K - p(t)) - y(t)],$$

where $\tilde{R}_0 \equiv R_0/\beta$ and $K \equiv m\left(1 - \dfrac{1}{R_0}\right)$. Note that, since $R_0 > 1$ by assumption, K must be strictly positive. We can now proceed with the optimisation problem.

3.   The optimisation problem

The socially efficient vaccination program maximizes the objective function

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[8] In Appendix A we discuss how the size of σ relates to the parameters in the original Anderson and May (1991) model.

$$(5) \qquad W = \int_0^T e^{-rt}[-c(p(t)) - by(t)]dt \,,$$

where *by(t)* is the cost at time *t* of having a proportion *y(t)* of the population infected, *c(p(t))* is the cost at time *t* of vaccinating a proportion *p(t)* of persons per unit of time (e.g., per year), and *T* is the length of the vaccination program, which may be finite or infinite. If the disease is eradicated, *T* will be finite, and the integral of social welfare from *T* to infinity will be zero and so can be ignored.

We assume that *c(0)* = 0 and that *c(p)* is strictly increasing and strictly convex. In Sections 5 and 6 we consider special cases of linear and quadratic costs, respectively. Note that *c(p)* includes more than just the costs of vaccine and of administering the vaccine. It also includes the costs of any side effects. The latter cost can be significant. For every million people given the smallpox vaccine, for example, a few will die and many others will suffer severe reactions. Similarly, and as explained in Section 7, the oral polio vaccine can cause paralysis in a very small percentage of cases. Worse, it can circulate in the community, infecting other susceptible persons. When a disease is prevalent, these associated effects are little noticed, but when control becomes very high, they become more prominent.

The government's problem is to maximize (5) subject to (4) and the additional constraints:

$$(6) \qquad p(t) \geq 0 \,,$$

$$(7) \qquad y(t) \geq 0 \,,$$

$$(8) \qquad y(0) > 0 \text{ given,}$$

and

$$(9) \qquad y(T) = 0 \,.$$

Except where stated otherwise, we shall assume $y(0) = \tilde{R}_0 K$. This is the steady state stock of infections when $p = 0$ (see eq. (4)).

Eq. (9) is of particular interest. It says that, once the disease is eradicated, there will be no more infections—and, therefore, no further need to vaccinate. The time T at which eradication is achieved is endogenous, determined as part of the solution to the optimisation problem. As noted before, T may be infinite, implying that it is not optimal to eradicate the disease. In our formal mathematical treatment, however, it shall prove convenient to assume that T is finite, i.e. that the choice of T is restricted to $T \in [0, \tau]$, where τ is very large (e.g., 5 million years). If we find that $T = \tau$ is optimal, this can be interpreted as saying that T is infinite.

4.   The optimal policy

Taking the shadow price $\alpha(t)$ associated with (4) to be positive, the current value Hamiltonian may be written as

(10) $H = -c(p(t)) - by(t) - \alpha(t)\sigma[\tilde{R}_0(K - p(t)) - y(t)]$.

Along the optimal program, the shadow price $\alpha(t)$ obeys the following differential equation (dropping the time argument when this causes no confusion):

(11) $\dot{\alpha} = r\alpha + \dfrac{\partial H}{\partial y} = (r + \sigma)\alpha - b$.

At any point in time, $p(t)$ maximizes the Hamiltonian. For $p(t)$ positive, maximization requires

(12) $c'(p) = \sigma\tilde{R}_0\alpha$.

Along the optimal programme, vaccination should be chosen at each instant in time such that marginal cost equals marginal benefit—the latter being equal to the shadow

value on infections, α, times the change in the number of infections attributable to a small change in the vaccination rate. Eq. (12) defines an increasing function, $p(\alpha)$, for $\alpha \geq c'(0)/\sigma\tilde{R}_0$. For $\alpha \leq c'(0)/\sigma\tilde{R}_0$, the constraint $p(\alpha) = 0$ applies.

From (4) and (12) we have

(13) $y = \tilde{R}_0(K - p(\alpha))$ for $\dot{y} = 0$

and

(14) $\alpha = \dfrac{b}{r+\sigma}$ for $\dot{\alpha} = 0$.

In $y - \alpha$ space, the $\dot{\alpha} = 0$ line is horizontal, whereas the $\dot{y} = 0$ line intersects the α-axis at $p(\alpha) = K$, decreases until $\alpha = c'(0)/\sigma\tilde{R}_0$, and then becomes vertical at $y = \tilde{R}_0 K$. Denote the intercept of the $\dot{y} = 0$ line α^0. Setting $p = K$, (12) gives:

(15) $\alpha^0 = \dfrac{c'(K)}{\sigma\tilde{R}_0}$.

There are two qualitatively different cases to consider. In the first, the $\dot{\alpha} = 0$ line lies above the $\dot{y} = 0$ line. In the second, these lines intersect in the interior. When solving both cases, we start by analysing the optimal solution assuming that T is given. Later we solve for the optimal value of T.

4.1   Case 1

We first consider the case where the $\dot{\alpha} = 0$ line lies above the $\dot{y} = 0$ line, i.e. where α, defined by (14), is higher than α^0, defined by (15). Rearranging gives:

$$(16) \quad \frac{b\sigma\tilde{R}_0}{r+\sigma} \geq c'(K).$$

Before proceeding with the mathematics, consider the economic implications of this condition. Begin at $t = 0$ with $y > 0$ given. Now, set $p = K$ and hold the vaccination rate at this level indefinitely. The marginal cost of this vaccination policy (at every moment in time) is given by the RHS of (16). From (4) we know that pursuit of this policy implies $y \to 0$ as $t \to \tau$ (recall that τ is very, very large). Eq. (4) also tells us that the instantaneous effect of the policy is to reduce the number of infections by $\sigma\tilde{R}_0$. Each infection saved yields a social benefit, b, and so the marginal, instantaneous benefit of this vaccination policy is $b\sigma\tilde{R}_0$. The full marginal benefit is larger, however, because the effect of vaccination is long lasting. From (4) we know that, when $p = K$, $\dot{y}/y = -\sigma$. Hence, the infections saved by this policy (at each instant in time) degrade at constant rate σ. Of course, the economic benefit is also discounted (at rate r). The marginal benefit of following this policy at each instant in time is thus $\int_0^T b\sigma\tilde{R}_0 e^{-(r+\sigma)t} dt$. As already noted, pursuit of this policy eradicates the disease only in the limit (that is, the disease is not eradicated). Solving the integral (for $T \to \tau$, which is close to ∞) yields the LHS of (16): the present value marginal benefit of vaccination for this policy.

Inequality (16) is a kind of reference condition. For consider a small deviation in the policy described above. Suppose in particular that at some date the vaccination rate is increased very slightly above $p = K$ for a very short period of time and then set equal to K again. The cost of this one-time deviation will be approximately equal to the RHS of (16). The benefit, however, will strictly exceed the LHS of (16) because this tiny, one-time increase in vaccination will cause the disease to be eradicated in finite time. Hence, (16) is only a sufficient condition for eradication to be optimal. It is not necessary.[9]

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[9] Intuitively, eradication implies that the infections saved from a policy deviation do not degrade. Hence, it might seem that a necessary condition for eradication to be optimal should be given by (16) but with σ removed from the denominator. We are unable to prove this for the general model, but our analyses in Sections 5 and 6 of two special cases confirm this intuition.

Case 1 is illustrated in Figure 1. The optimal development of $y(t)$ and $\alpha(t)$ (and thus of $p(t)$) depends on the exogenously given value of T. We have drawn trajectories for three different values of T. For each trajectory we have assumed that $y(0) = \tilde{R}_0 K$ (this gives the steady state value of y when $p = 0$). The top trajectory in Figure 1 is for a "small" T, denoted T^1. Along this trajectory, $p(t)$ increases over time. The middle trajectory is for a value of T which implies that $p(t)$ must be constant over time. Finally, the bottom trajectory is for a "large" T, denoted T^2. Along this trajectory, $p(t)$ declines over time. For $y(0)$ given, as the value of T increases, the terminal value of the shadow price, $\alpha(T)$, falls. It approaches α^0 as T approaches infinity (strictly speaking, τ).

The value of the Hamiltonian at time T, denoted $H(T)$, follows from (9) and (10):

$$(17) \quad H(T) = -c(p(T)) - \alpha(T)\sigma \tilde{R}_0[K - p(T)].$$

If we can find a T^* such that $H(T) \geq 0$ for $T \leq T^*$ and $H(T) \leq 0$ for $T \geq T^*$, then this will be an optimal solution to our optimisation problem when T is endogenous.[10]

In Appendix B we show that $H'(T) > 0$, and that the value of p making $H(T) = 0$, denoted p^*, is given by

$$(18) \quad c'(p^*) = \frac{c(p^*)}{p^* - K}.$$

p^* thus depends on both the cost function and K (that is, on m and R_0).

The corresponding value of α, denoted α^*, is given by (see (12))

$$(19) \quad \alpha^* = \frac{c'(p^*)}{\sigma R_0}.$$

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[10] See, e.g., Theorem 1 in Seierstad (1988). Notice that the Hamiltonian given by (10) with $p(\alpha)$ inserted is linear, and thus concave, in y.

The value of α^* depends on the factors determining p^* and on σ. Since $p^* > K$, it follows from (15) and (19) that $\alpha^* > \alpha^0$. Since all paths leading to $\alpha^* > \alpha^0$ result in eradication, it follows that inequality (16) is a sufficient condition for eradication to be optimal, confirming the economic intuition given earlier.

If we can find a trajectory for $(y(t), \alpha(t))$ satisfying the differential equations (4) and (11), starting at $(y(0), \alpha(0))$ where $y(0)$ is given by (8) and ending at $(0, \alpha^*)$ at some time point T, then this T, denoted T^*, is the optimal end point.

The optimal trajectory in Figure 1 is thus the one that terminates at $\alpha(T) = \alpha^*$. All three of the paths shown in Figure 1 are potential candidates. Which trajectory is optimal depends on the value of α^*, and thus on the factors that determine α^*.

4.2   Case 2

Assume now that the inequality in (16) is reversed. Since (16) is a sufficient condition for eradication to be optimal, we should expect that, for Case 2, eradication may or may not be optimal. We confirm this intuition below.

Case 2 is illustrated in Figures 2 and 3. Figure 2 assumes that there exists a trajectory for $(y(t), \alpha(t))$, starting at $(y(0), \alpha(0))$, where $y(0) = \tilde{R}_0 K$ and $\alpha(0) > b/(r + \sigma)$, and ending at $(0, \alpha^*)$ at $t = T^*$. This case is thus similar to Case 1. The only difference is that we can now be sure that $p(t)$ increases over time.

It is also possible that all trajectories starting at $y(0) = \tilde{R}_0 K$ and $\alpha(0) > b/(r + \sigma)$ reach $y = 0$ at a value of α that exceeds α^*. Under these conditions, no trajectory of the type illustrated in Figure 2 will exist, and eradication in finite time will not be optimal.

However, there will always exist an unstable stationary point $(y^\infty, b/(r+\sigma))$, where

$$(20) \quad y^{\infty} = \tilde{R}_0 \left[K - p \left(\frac{b}{r+\sigma} \right) \right].$$

The trajectory starting at this point and moving in a northwest direction intersects the α-axis at a point labelled α^{∞} in Figure 3. If a trajectory of the type drawn in Figure 2 does not exist, we have $\alpha^{\infty} > \alpha^*$. Under these conditions, and taking $y(0) = \tilde{R}_0 K$, we have $\alpha(T) > \alpha^{\infty} > \alpha^*$ for all $T \in [0, \tau]$. This means that $p(T) > p^*$ for all $T \in [0, \tau]$. From Appendix B it follows that $H(T) > 0$ for all $T \in [0, \tau]$. The optimal end point, therefore, is τ (in practical terms, infinity).[11] The optimal solution is to set $p(t) = b/(r+\sigma) \, \forall t$, arriving at y^{∞} asymptotically.[12] That is, the disease is controlled but not eradicated. It is clear from Figure 3 that p will be smaller, and y^{∞} closer to $y(0) = \tilde{R}_0 K$, the smaller is $b/(r+\sigma)$ and the larger is $c'(0)/\sigma \tilde{R}_0$. If the marginal cost of vaccination exceeds the social marginal benefit when $p = 0$—that is, if $c'(0) > b\sigma \tilde{R}_0/(r+\sigma)$—then the optimal policy will be to set $p = 0$ always (technically, before τ is reached), unless eradication in finite time is optimal.

To sum up, we have thus far established a sufficient condition for eradication to be optimal, and we have characterized the other possible qualitative solutions. To derive more specific results—in particular, a necessary and sufficient condition for eradication to be optimal—we will have to work with explicit cost functions. We turn to this task in Sections 5-6, but first it will prove helpful to consider the effect of the initial conditions on the results developed thus far.

4.3   Initial conditions

To this point we have assumed that the starting value of y is given by y's stationary value when $p = 0$—that is, $\tilde{R}_0 K$. What would be the optimal policy at an early stage of a new disease when the initial infection rate is substantially below $\tilde{R}_0 K$?

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[11] See e.g. Theorem 1 in Seierstad (1988).

[12] Strictly speaking, since τ is finite, the trajectory lies infinitesimally above the trajectory going from $(\tilde{R}_0 K, b/(r+\sigma))$ to $(0, \alpha^{\infty})$ via $(\alpha^{\infty}, b/(r+\sigma))$, lying close to the latter point most of the time.

Plainly, if eradication were optimal when $y(0) = \tilde{R}_0 K$, then it will also be optimal when $y(0) < \tilde{R}_0 K$. Indeed, the optimal program will require that vaccination proceed along the same optimal trajectory as derived above (that is, the optimal trajectory corresponding to the starting value $y(0) = \tilde{R}_0 K$). The only difference is that, since the starting value of y is different, the starting value of p must also be different. Our analysis thus applies equally well to a situation in which a disease has been controlled previously as to a situation in which a disease has not been controlled at all.

The initial conditions only really matter when eradication would not be optimal for $y(0) = \tilde{R}_0 K$. If $y(0)$ is small enough (relative to y^∞), then it can be shown that eradication will be optimal. This possibility is illustrated in Figure 4, which differs from Figure 3 only with respect to the initial condition. For the starting values ($y(0)$, $\alpha(0)$) in Figure 4, the trajectory reaches (0, α^*), and so it is optimal to eradicate the disease at some time $T^* < \tau$. The reason that eradication will be optimal when the initial rate of infection is low is not that fewer people need to be vaccinated at any given time.[13] The reason is that people need be vaccinated for a shorter period of time.

The new disease SARS (severe acute respiratory syndrome), we now know, emerged in late 2002 in China. In March 2003, the World Health Organization issued a global alert, and countries immediately began taking measures to control the disease. Some scientists argued that this was not enough, however, that the opportunity to eradicate the disease should be seized before SARS had a chance to become established. As Burke (2003) put it, "epidemic-control efforts should not simply be maintained, but doubled, and redoubled again." The rationale for moving quickly was that there existed but a short epidemiological window during which SARS could be readily distinguished from influenza. Wait too long, or act too passively, and eradication might cease to be feasible. This paper points to a further rationale: While a short,

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[13] In our model, control is achieved by means of mass vaccination. Only in a model with heterogeneous mixing would a strategy like "ring vaccination" work, making it possible to isolate infected persons and to vaccinate only those susceptible persons who came into contact with infected individuals before quarantine.

sharp response may be optimal at the early stage of the disease, a sustained effort at eradication may not be optimal after the disease has become established.[14]

5. The special case of a linear cost function

To get sharper results, it will prove useful to consider the case of a linear cost function. Specifically, let

(21) $c(p) = cp$,

where c is a positive parameter. To get a mathematically meaningful solution to our maximization problem, we assume instead of (6) that

(22) $p(t) \in [0, P]$,

where P is some large value (certainly large enough to make eradication possible). We shall in particular consider the limiting case in which $P \to \infty$.

Instead of (12) we now get

$$p(t) = 0 \text{ for } \alpha(t) < \frac{c}{\sigma \tilde{R}_0},$$

(23)

$$p(t) = P \text{ for } \alpha(t) > \frac{c}{\sigma \tilde{R}_0}.$$

The optimal vaccination program thus reaches the optimal steady state infection rate as quickly as possible. This result is not very surprising; we should expect to obtain a most rapid approach solution for the linear model. What is more surprising, however, is that, for the linear model, there are only two optimal steady states. It is optimal

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either not to vaccinate at all or to eradicate. A policy of disease control (short of eradication) is never optimal.

What is the reason for this result? Recall from Section 2 that our simplification of the dynamics implies a constant marginal benefit of vaccination for any level of control short of eradication. With the linear model, marginal costs are also constant. Hence, if it is better to vaccinate many persons than one fewer, then it must also be better to vaccinate one person than none. We know that eradication is better than a policy of vaccinating many persons. For the linear model, therefore, eradication must also be better than a policy of vaccinating even one person. However, eradication need not be welfare superior to a policy of zero control. Hence, with the linear model, only one of two extreme outcomes will be optimal: eradication or no control. It is important to emphasize that this result follows not only from the assumption about costs, but also from the way in which we have represented the dynamics of infection, as noted in Section 2. Even with linear costs, positive vaccination short of eradication may be optimal if the number of follow-on infections prevented by each vaccination were decreasing in the vaccination rate.

The $\dot{y} = 0$ line is now horizontal as it meets the vertical axis at $\alpha^0 = c/\sigma\widetilde{R}_0$. Inserting $H(T^*) = 0$ into (17) gives the value of $\alpha(T^*)$, i.e. α^*:

$$(24) \quad \alpha^* = \frac{cP}{\sigma\widetilde{R}_0(P-K)} .$$

As in the general case, $\alpha^* > \alpha^0$. We also have $\alpha^* \to \alpha^0$ as $P \to \infty$.

When $\alpha^0 < b/(r+\sigma)$, the linear model yields an outcome qualitatively identical to the general model: eradication is optimal. The only noteworthy difference is that, for the linear model, eradication is achieved immediately for the limiting case of $P \to \infty$.

The more interesting case, drawn in Figure 5, arises when $\alpha^0 > b/(r+\sigma)$. As was shown for the general case (see Figure 3), we now have an unstable stationary point $(y^\infty, b/(r+\sigma))$. The difference is that, for the linear model, y^∞ will always equal

$y(0) = R_0 K$; partial control is never optimal. The trajectory rising from this point intersects the α-axis at α^∞. Assume first that $\alpha^\infty < \alpha^*$ (we have not illustrated this case). Then there will exist a trajectory for *(y(t), α(t))* starting at *(y(0), α(0))*, where $y(0) = \widetilde{R}_0 K$ and *α(0) > b/(r+σ),* and ending at *(0, α*)* at *t = T**. This solution is akin to the general case shown in Figure 2. For the linear model, *p(t) = P* for all *t* ∈ *[0,T*]*. For the case illustrated in Figure 5, $\alpha^\infty > \alpha^*$. Given $y(0) = \widetilde{R}_0 K$, $\alpha(T) > \alpha^\infty > \alpha^*$ for all $T \in [0, \tau]$. It is easily verified from (17) and (23) that this implies *H(T) > 0* for all $T \in [0, \tau]$. The optimal end point is therefore τ—in practical terms, infinity. [15] The optimal policy is never to vaccinate (strictly speaking, when τ is finite, we should have *p(t) = 0* until just before τ, after which *p(t) = P*).

The limiting case of $P \to \infty$ yields a particularly useful result (the proof is given in Appendix C): *For the linear model, and taking $P \to \infty$, eradication is optimal if and only if*

$$(25) \quad c < b\sigma \widetilde{R}_0 / r.$$

Moreover, if condition (25) holds, and if a policy of setting $P \to \infty$ is feasible, then eradication should be achieved instantaneously.[16]

6.   The special case of a quadratic cost function

For the general model, three qualitatively different outcomes may be optimal: no vaccination, control short of eradication, and eradication. With a linear cost function, only the two extreme outcomes of no vaccination and eradication are optimal. For the quadratic cost function considered in this section (in which marginal cost is near zero for the first vaccination), the outcome of no vaccination will not be optimal. Our aim

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[15] See Theorem 1 in Seierstad (1988).

[16] It is interesting to compare this result with the corresponding condition given in Barrett (2003). Setting *n* = α = 1 and $p^u = 0$ in Barrett's eq. (7) gives the result that eradication is optimal if $bR_0 / r \geq c$. This is equivalent to the condition given above once we set $\sigma = \beta = 1$. In Barrett's (2003) model, p^u need not equal zero because the social benefit of vaccination is non-linear in the vaccination rate.

here is thus to derive conditions under which it will be optimal to eradicate rather than to control a disease.

The cost function is

$$(26) \quad c(p) = \frac{g}{2}p^2,$$

where $g > 0$, giving a marginal cost $c'(p) = gp$. For this case, the function $p(\alpha)$ defined by (12) gives

$$(27) \quad p(t) = \frac{\sigma \tilde{R}_0}{g}\alpha(t).$$

Moreover, it follows from (15), (18), and (19) that, for the present case,

$$(15') \quad \alpha^0 = \frac{gK}{\sigma \tilde{R}_0},$$

$$(18') \quad p^* = 2K,$$

and

$$(19') \quad \alpha^* = \frac{2gK}{\sigma \tilde{R}_0}.$$

Since $c'(0) = 0$, it is never optimal not to vaccinate. Moreover, we know from (16) that, if

$$(28) \quad gK < \frac{b}{r+\sigma}\sigma \tilde{R}_0,$$

then eradication will be optimal. The interesting case is when the inequality in (28) is reversed, in which case eradication may or may not be optimal. For the general

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model, we were only able to derive a sufficient condition for eradication to be optimal. For the specific quadratic function, however, we can give a necessary and sufficient condition. Appendix D derives explicit solutions for the differential equations (4) and (11), and solves for the conditions under which an optimal trajectory leading to (0, α*) exists. These calculations imply that: *For the quadratic model, eradication is optimal if and only if*

$$(29) \quad gK < \frac{b\sigma\widetilde{R}_0}{r}.$$

This condition is very similar to the condition for eradication for the case of a constant unit cost (see Section 5). The only difference is that the relevant vaccination cost now is not the unit cost (constant for all vaccination rates), but the marginal cost at the minimum vaccination level necessary to achieve eradication (i.e., K; see (4)).

It is significant that the optimality condition for eradication should depend on this marginal cost, because the literature on vaccination routinely assumes (implicitly, at least) constant average costs. When R_0 is large, K will be large: a sizable proportion of the population must be vaccinated in order to reduce incidence to zero. Expanding coverage, however, is costly. It means reaching people in remote areas, the homeless, people with compromised immune systems, and people with religious objections to vaccination. The marginal cost of eradication can be substantially greater than the average cost.

7. Application to polio eradication

Our aim has been to characterize the optimal disease eradication program. However, in the course of doing so we have also derived cost-benefit rules for eradication, and it is tempting to apply these to a real problem. We do so below.

The global polio eradication initiative—according to the World Health Organization (2001: 1), "the largest public health initiative in history"—aims to eradicate polio by 2005. Bart, Foulds, and Patriarca (1996) and Khan and Ehreth (2003) have shown that

the program promises the world a benefit in excess of cost, assuming that eradication can be achieved by means of the oral live-attenuated polio vaccine (OPV) and that vaccination can cease after the wild virus has been eradicated. In this section we use our model as a check on these studies, and to evaluate an important, alternative scenario.

OPV has a number of advantages: it is inexpensive and easy to administer; it stimulates local immunity in the intestines, preventing spread of the disease; and when the vaccine virus is shed in areas with poor hygiene and sanitation, it immunizes the community. OPV also has one disadvantage: in a very small number of cases, the vaccine can cause paralysis, either in vaccinated persons (vaccine-associated paralytic polio or VAPP) or in susceptible individuals in the community (circulating vaccine-derived polioviruses or cVDPV). VAPP is especially problematic when vaccination coverage is high, for then the risk of VAPP can exceed the risk of infection by the wild virus. cVDPV, by contrast, is especially problematic when vaccination stops, for then susceptible persons are vulnerable to infection by cVDPV. Though fewer than 100 cases of cVDPV have been documented to date, there is a chance that cVDPV could become endemic—making it necessary to continue OPV vaccination indefinitely (WHO, 2003: 16). Our analysis has shown that maintaining a high level of vaccination is not optimal when eradication is feasible. Of course, it may be optimal to maintain vaccination at a high level when eradication (including eradication of vaccine-derived virus) is *not* feasible, but the economics of the polio eradication initiative will plainly be very different if OPV-vaccination must be continued.

Our framework is not suited to evaluating the decision to maintain high vaccination indefinitely, but it can be used to evaluate the eradication options. One option is to stop OPV vaccination following a synchronized, global pulse campaign in which very high rates of coverage are achieved over a very short period of time. This would reduce the number of susceptible individuals everywhere, and so reduce the risk of cVDPV spreading. However, this approach would still be a gamble; it may not

prevent the disease from remerging.[17] Another option would switch from OPV to the inactivated polio vaccine (IPV). IPV has one great advantage: it is a killed poliovirus, and so cannot cause polio. However, IPV also has disadvantages: it is more expensive than OPV and must be injected; it does not prevent transmission by vaccinated individuals; and it does not spread immunity throughout the community.

Use of either vaccine, even for purposes of control, implies a trade-off. In areas with excellent hygiene, sanitation, and health infrastructure, and with high vaccination coverage, OPV has only a cost advantage over IPV, whereas IPV protects vaccinated individuals from the risk of VAPP. After polio had been eliminated from the United States, continued use of OPV caused about 10 cases of VAPP a year—a small number, perhaps, given the extent of vaccination coverage, but a sufficient risk to impel the US recently to switch from OPV to IPV, despite the higher cost.[18] In poor countries, OPV remains the vaccine of choice, mainly because of cost and the benefits of community-wide protection.

Is polio eradication optimal? In our framework, this is roughly the same as asking if conditions like (25) and (29) hold for polio. Let us see. From Appendix A (eq. (A.7)), we know that $\sigma\widetilde{R}_0$ can be approximated by $m(R_0-1)/(m+v)$. For polio, $R_0 \approx 6$ (Anderson and May, 1991: 70). The infectious period for polio lasts about 14-20 days (Anderson and May, 1991: 31), and the parameter v is approximately equal to the inverse of this duration (Anderson and May, 1991: 125).[19] Assuming that the duration of infection is 20 days, and taking time units to be years, implies that $v \approx 18.25$. Though we take these parameters to be the same for all countries, the remaining parameter, m, representing the birth and death rate, can vary. We take it that $m = 0.01$ for rich countries and $m = 0.03$ for poor countries, though our results will not be sensitive to these values.[20]

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[17] To make matters worse, vaccine-derived polio virus can be excreted for years by people with immune deficiencies.

[18] There are about 250-500 cases of VAPP worldwide every year (WHO, 2003: 16). For a cost-benefit analysis of the decision by the US to discontinue OPV, see Miller *et al.* (1996).

[19] Our model does not include a latent period, but for polio this is short—about 1-3 days (Anderson and May, 1991: 31). This means that a person who acquires poliovirus ceases to be infectious after about 15-23 days.

[20] Over the period 1980-2001, population growth was 0.7 percent in the high-income countries and 2.7 percent in sub-Saharan Africa (World Bank, 2003: 40).

The literature only offers estimates of average costs, and these are assumed constant. Khan and Ehreth (2003: 703) give cost estimates for two kinds of vaccination program, routine immunization and mass vaccination on national immunization days. For poor (rich) countries, these are $1.22 ($11.50) and $0.25 ($5.64) per dose, respectively, using OPV. We take the marginal cost of OPV to be the higher of these estimates: $1.22 per dose in poor countries and $11.50 per dose in rich countries. Four doses of vaccine are required for immunity, and so our estimates for c are $46 in rich countries and $4.88 in poor countries.

According to Miller *et al*. (1996: 969), IPV is about 35 percent more expensive than OPV in the United States. In poor countries, IPV is even more expensive relative to OPV, partly because IPV requires a syringe and must be administered by a trained health worker. According to the polio eradication initiative's web page, IPV is more than 5 times as costly as OPV, leaving aside administration costs.[21] We assume that a full, four-dose course of IPV costs $25 in poor countries and $62 in rich countries.

The literature does not give true estimates for b, the welfare cost of paralytic polio. Khan and Ehreth (2003: 703) only give estimates of medical care costs, assumed to equal $420 in poor countries and $25,000 in rich countries. These probably underestimate the true costs substantially. Miller *et al*. (1996: 968) use a very different estimate in their study of the decision to switch from OPV to IPV in the United States: $1.2 million, the compensation awarded to VAPP victims in the United States. Rather than use these numbers in our analysis, we solve for the value of b that just makes eradication optimal, and then discuss whether, given these values, eradication really is optimal.

There is just one more adjustment to make. Most people who become infected with poliovirus do not show symptoms. About one in 200 suffer paralysis (WHO, 2002). We therefore take the value of b to be 0.005 times the cost of a single case of polio paralysis.

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[21] See http://www.polioeradication.org/vaccines/polioeradication/all.

We can now proceed with our calculations.[22] Letting $r = 0.03$, Table 1 shows the value of b that just makes eradication optimal, for rich and poor countries. [23]

TABLE 1

Optimal Polio Eradication Cut-Off Values for b

	Rich	Poor
OPV	$100,795	$3,568
IPV	$135,854	$18,280

Based on the assumptions underlying these calculations, and assuming that b should represent medical care costs (see below), polio eradication would seem not to be optimal, even assuming that use of OPV can be discontinued. This is a qualitatively different conclusion than appears in the earlier literature. Bart, Foulds, and Patriarca (1996) find that the benefits of eradication exceed the costs for the world as a whole, compared with a policy of routine immunization; and Khan and Ehreth (2003) find that the total medical cost savings of eradication exceed the total cost. However, we would caution making a direct comparison between these studies and ours. Among other differences, our model compares an optimal non-eradication outcome with the eradication alternative. The alternatives investigated in these earlier studies are not chosen optimally.

As well—and as recognized by the authors of both of the above studies—medical care costs are not appropriate measures for b. The welfare costs of infection are likely to be much higher. Plainly, the cut-off value of b for the rich countries is less than a tenth of the compensation awarded to VAPP victims in the US, implying that eradication is

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[22] In the analysis that follows, we ignore the cost of VAPP associated with OPV vaccination.

[23] Since we only have average cost data, and no information about how costs vary with the vaccination rate, we use inequality (25), after substituting (A7). OPV calculations: for rich countries we have $46 < 0.005 * b * 0.01 * (6-1)/[(18.25 + 0.01) * 0.03]$; for the poor, $4.88 < 0.005 * b * 0.03 * (6-1)/[(18.25 + 0.03) * 0.03]$. IPV calculations: for the rich we have $62 < 0.005 * b * 0.01 * (6-1)/[(18.25 + 0.01) * 0.03]$; for the poor, $25 < 0.005 * b * 0.03 * (6-1)/[(18.25 + 0.03) * 0.03]$. The values in Table 1 are found by solving these inequalities.

optimal from the perspective of the rich countries. The cut-off value for *b* for poor countries is very low by comparison, but the opportunity costs of polio eradication are different in poor countries. The value per disability life year (DALY) avoided by polio eradication exceeds $900.[24] Many other health interventions in poor countries can save a DALY for a fraction of the cost (see, for example, Jamison *et al*., 1993).

What does this mean? It means that, if all other countries eliminated polio, every rich country also would want to do so. The poor, however, would not. Of course, the gain for the rich countries may exceed the loss for the poor, so that, with suitable transfers, polio eradication may make all countries better off. Indeed, this presumably explains why the polio eradication initiative was undertaken in the first place, and why the rich countries and other donors are financing the (incremental) costs of polio elimination in the poor countries. At the same time, our results are consistent with the ambivalent attitude towards polio eradication expressed by public health experts.[25]

What are the economics of eradication using IPV? Following the same approach as above, but using the higher costs of vaccinating with IPV, we find that eradication by IPV is less attractive all around but remains a good deal for the rich countries (several industrialized countries have already switched to IPV). For the poor countries, by contrast, eradication by IPV is almost certainly uneconomic. Still, eradication by IPV may be globally optimal. To know for sure, however, a full cost-benefit analysis should be undertaken. Our analysis is only indicative.[26]

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[24] According to Khan and Ehreth (2003: 704), over the period 1970-2050, polio immunization would save 6.89 million polio cases and 4.396 million DALYs in Africa. Khan and Ehreth (2003: 703) assume that 10.27 percent of all polio cases result in paralysis. Hence, the implied number of DALYs lost per case of paralytic polio is about 4.396/(6.89*0.1027) = 6.2. The cut-off value of a case of paralytic polio is thus about $5,947, or $5,947/6.2 = $959 per DALY.

[25] See, in particular, Taylor, Cutts, and Taylor (1997) and Sutter and Cochi (1997).

[26] In addition to performing sensitivity analysis, our analysis might be extended by incorporating further adjustments. For example, there is typically wastage in the use of vaccine. As well, vaccination efficacy is typically less than full. Making adjustments for wastage and efficacy would effectively increase vaccination costs. Finally, we suspect that R_0 may be larger in poorer countries with weaker standards of hygiene and sanitation. Increasing the value of R_0 in the poor countries would obviously lower the cut-off value of *b*.

8.   Conclusions

Our analysis applies to infectious diseases for which eradication is epidemiologically feasible. At a minimum, these include global diseases like polio, measles, and rubella (Knobler, Lederberg, and Pray, 2002). We have shown that eradication, when feasible, will often be preferable to control—and will always be preferable to high rates of control. We have also shown that rapid progress towards eradication will usually be preferred. Only when vaccination costs increase substantially with the rate of vaccination should a slower course be followed.

An implication of our analysis is that, when rich countries are observed to set a high level of control, this can be taken to be an economic indicator of eradication being possibly optimal. Plainly, if a country would eliminate a disease even when eradication is infeasible (because of the risk of imports), then it would certainly eradicate the disease if eradication were feasible—eradication would cost no more than elimination but offer a huge dividend in avoided future vaccination costs. For the poor countries, the calculus is likely to be different. As indicated by our analysis of polio eradication, transfers from the rich to the poor are almost certain to be needed to effect global eradication of this disease. A full, global cost-benefit analysis is needed to determine whether eradication is a good deal overall, but our optimality conditions provide a basis for making a first assessment of the economics of eradication versus control.

We end with a final observation. It is routine in health economics to rely on average benefit and cost estimates. For most policy analysis, this is probably satisfactory. For eradication, it is not. Eradication is an extreme goal, and our paper shows that our analysis of an eradication program needs to begin from the perspective of the program's end. Eradication only succeeds if the last carrier of the disease is isolated, and the persons with whom he or she may have come into contact are vaccinated. It is fitting that our optimality rule should also focus on this last case.

Appendix A

The relationship between our dynamic specification and the specification used by AM [Anderson and May (1991)]:

At any point in time, the proportion of the population that is susceptible is $x(t)$, while the proportion that is infected is $y(t)$. The remaining proportion of the population, $1 - x(t) - y(t)$, is immune.

The interpretation of (1) is that the gross increase in the proportion of susceptibles is equal to the birth rate (m), while the gross reduction in the proportion of suscepticles is the sum of those who die naturally ($mx(t)$), those who become infected ($\lambda(t)x(t)$), and those who become immune due to vaccination ($p(t)$).

In addition to equation (1), AM assume that the proportion of infected persons develops according to

$$(A1) \quad \dot{y}(t) = \lambda(t)x(t) - (v + m)y(t).$$

The interpretation of (A1) is that those who have become infected either die naturally ($my(t)$) or recover into the immune class ($vy(t)$).

The "force of infection", $\lambda(t)$, is the per capita rate of acquisition of the infection. In other words, $\lambda(t)\Delta t$ represents the probability that a given susceptible host will become infected in a small time interval Δt. AM argue that with homogenous mixing, $\lambda(t) = \beta y(t)$, where β is a transmission parameter that depends on various epidemiological, environmental, and social factors. Inserting $\lambda(t) = \beta y(t)$ into (A1) and defining the basic reproductive rate of the microparasite (according to "Type II survival"; see AM: 75) by

$$(A2) \quad R_0 = \frac{\beta}{v + m},$$

we can rewrite (A1) as (2).

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The differential equations (1) and (2) have a stationary state (for a constant p) given by $x^\infty = 1/R_0$ and $\lambda^\infty = m(R_0 - 1) - pR_0$. Starting at the stationary state, consider the effect of a small increase ε in the proportion of infecteds, and a corresponding reduction in the proportion of susceptibles. Since $\lambda(t) = \beta y(t)$, this implies that λ increases by $\beta\varepsilon$. Immediately after such an increase, it is straightforward to see from (2) that we get

(A3) $\dot{\lambda} = (v + m)\lambda^\infty R_0(-\varepsilon)$.

With the differential equation (3) we would instead get

(A4) $\dot{\lambda} = -\sigma\beta\varepsilon$.

Using (A2), it is clear that these two differential equations give the same value for $\dot{\lambda}$ if and only if

(A5) $\sigma = \lambda^\infty = m(R_0 - 1) - pR_0$.

The RHS of (A5) depends on p, so that the value of $\dot{\lambda}$ following from AM and from (3) cannot be the same for all p. In the numerical application in Section 7 we let σ be determined by (A5) with $p = 0$, i.e.

(A6) $\sigma = m(R_0 - 1)$.

Using (A2) and the definition of $\widetilde{R}_0$ given at the end of Section 2, it follows that

(A7) $\sigma\widetilde{R}_0 = \dfrac{m}{m + v}(R_0 - 1)$.

Appendix B

The eradication date when eradication is optimal:

Differentiating (17), remembering that $p(\alpha)$ maximizes H, and using the envelope theorem, we obtain:

$$(B1) \quad H'(T) = \sigma R_0 [p(T) - K] \frac{\partial \alpha(T)}{\partial T}.$$

Since $y(t)$ approaches zero at $t = T$, it follows from (13) that the term in square brackets in (B1) is positive. Moreover, $\alpha(T)$ is decreasing in T (see the discussion in Section 4.1). From (B1) it therefore follows that $H'(T) < 0$. Hence, if we can find a value T^* giving $H(T^*) = 0$, then this will be an optimal solution to our optimisation problem.

Using the notation $p^T = p(\alpha(T))$, and inserting (12) into (B1), gives

$$(B2) \quad H(T) = -c(p^T) + [p^T - K]c'(p^T).$$

The value of p^T (denoted p^*) which makes $H(T) = 0$ is given by (18). From (12) it follows that the corresponding value of α, denoted α^*, is given by (19).

The RHS of (B2) is increasing in p^T for $p^T > p^*$. For $p^T > p^*$, as is the case in Figure 3, we therefore must have $H(T) > 0$.

Appendix C

Proof that, for $P \to \infty$, if $c > b\sigma \tilde{R}_0 / r$, then the optimal policy is never to vaccinate, whereas if $c < b\sigma \tilde{R}_0 / r$, then eradication is optimal:

Recall from (23) that it will either be optimal to do nothing or to vaccinate at the maximum feasible rate. The payoff from not vaccinating is:

(C1) $W^{do\,nothing} = \int_0^\infty e^{-rt}[-by(0)]dt = -\dfrac{b}{r}y(0)$.

The payoff from immediate eradication (implying $T \to 0$) is:

(C2) $W^{eradication} = \int_0^T e^{-rt}[-cP]dt = cPT$.

As $P \to \infty$, the term including P will dominate the other terms in (4), so that

(C3) $\dot{y}(t) = -\sigma \tilde{R}_0 P$,

which implies

(C4) $y(T) = y(0) - \sigma \tilde{R}_0 PT$

or, since $y(T) = 0$,

(C5) $PT = \dfrac{y(0)}{\sigma \tilde{R}_0}$.

Substitution into (C2) gives

(C6) $W^{eradication} = \int_0^T e^{-rt}[-aP]dt = -c\dfrac{y(0)}{\sigma \tilde{R}_0}$.

A comparison of (C6) and (C1) proves the result.

Appendix D

Proof that, for the quadratic model, eradication is optimal if and only if $gK < b\sigma \tilde{R}_0 / r$:

Rather than derive conditions ensuring that $\alpha^* > \alpha^\infty$, our approach is to derive conditions under which an optimal path leading to $(0, \alpha^*)$ exists. To do this, it is useful to rewrite differential equations (11) and (4) as functions of the time variable h, which denotes the time remaining until T^* is reached. With this notation, (11) and (4) can be written as

(D1) $\alpha'(h) + (r + \sigma)\alpha(h) = b$, $\alpha(0) = \alpha^*$

(D2) $y'(h) - \sigma y(h) = A\alpha(h) + B$, $y(0) = 0$,

where

(D3) $A = \dfrac{(\sigma \tilde{R}_0)^2}{g}$

and

(D4) $B = -\sigma \tilde{R}_0 K$.

Solving (D1) gives

(D5) $\alpha(h) = \left(\alpha^* - \dfrac{b}{r+\sigma}\right)e^{-(r+\sigma)h} + \dfrac{b}{r+\sigma}$.

Inserting (D5) into (D2) and solving gives

(D6) $y(h) = e^{\sigma h}(J_1 + J_2) - e^{-(r+\sigma)h}J_1 - J_2$,

where

(D7) $J_1 = \dfrac{A}{(r+2\sigma)}\left(\alpha^* - \dfrac{b}{(r+\sigma)}\right)$

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and

(D8) $J_2 = \dfrac{Ab}{\sigma(r+\sigma)} + \dfrac{B}{\sigma}$.

For the situation described by Figure 2, *y(h)* is positive (for *h* > 0) and increasing as we travel backwards in time. By inspection of (D6), *y(h)* will be positive (for *h* > 0) if $J_1 + J_2 > 0$. Since $y'(h) = \sigma e^{\sigma h}(J_1 + J_2) + (r+\sigma)e^{-(r+\sigma)h}J_1$, and since J_1 is positive for the situation described by Figure 2, *y(h)* will be increasing if $J_1 + J_2 > 0$. Hence, for the kind of situation depicted in Figure 2, eradication will be optimal if and only if $J_1 + J_2 > 0$. Inserting *A* and *B* from (D3) and (D4) into (D7) and (D8) we find

(D9) $J_1 + J_2 > 0 \Leftrightarrow gK < \dfrac{b}{r}\sigma\widetilde{R}_0$,

which is the condition given in (29). Finally, note that, if (29) holds then (28) will hold, confirming that (29) is the necessary and sufficient condition for eradication to be optimal for the quadratic model.

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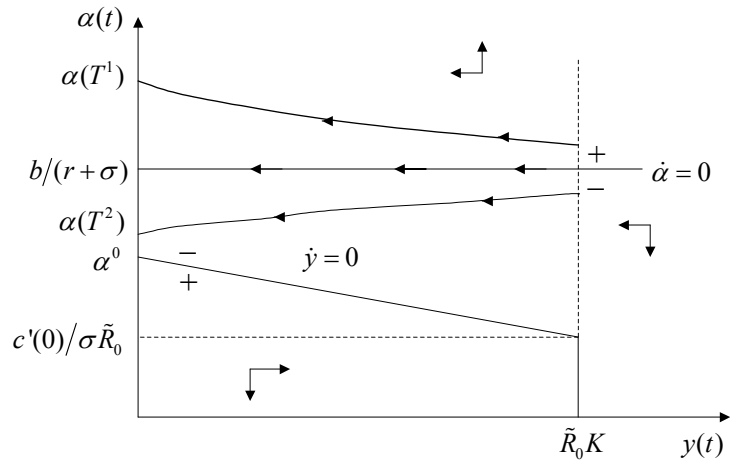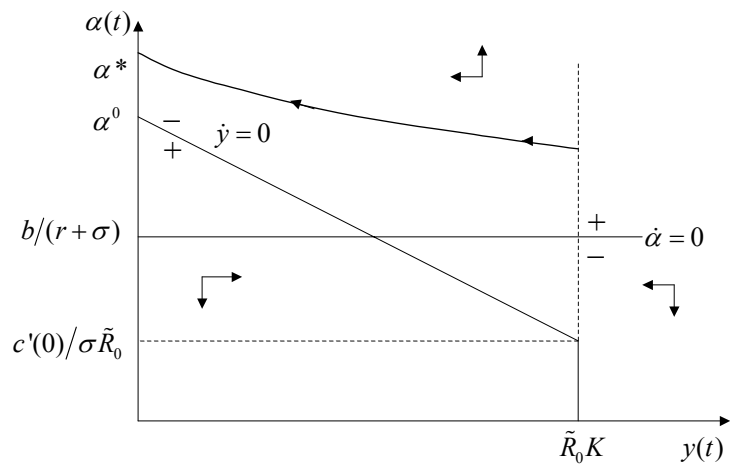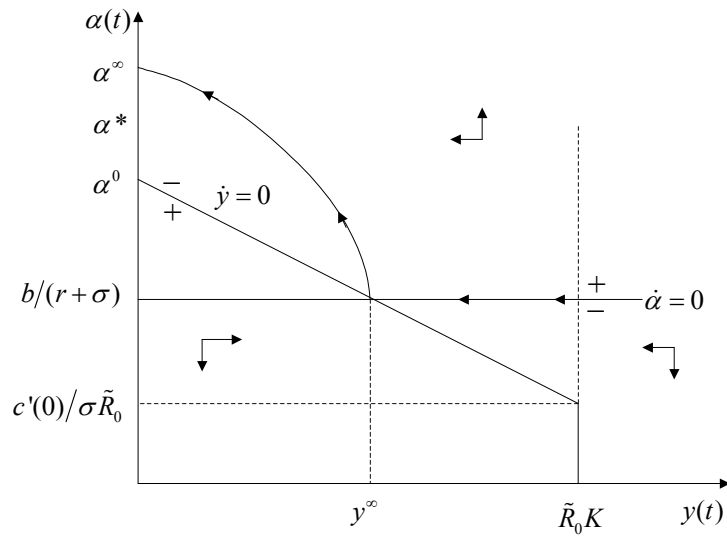
Figure 1

Figure 2

Figure 3

$\alpha(t)$

α^∞

α^*

α^0

$-$
$+$ $\dot{y}=0$

$b/(r+\sigma)$ $\dfrac{+}{-}$ $\dot{\alpha}=0$

$c'(0)/\sigma\tilde{R}_0$

y^∞ $\tilde{R}_0 K$ $y(t)$

Figure 4

$\alpha(t)$

α^∞

α^*

α^0

$\alpha(0)$

$b/(r+\sigma)$ $\dfrac{+}{-}$ $\dot{\alpha}=0$

$-$ $\dot{y}=0$
$+$

$c'(0)/\sigma\tilde{R}_0$

$y(0)$ y^∞ $\tilde{R}_0 K$ $y(t)$

Figure 5

$$\alpha(t)$$

$$\alpha^\infty$$

$$\alpha^*$$

$$\alpha^0 \equiv c/\sigma\tilde{R}_0 \quad \underline{} \quad \dot{y} = 0$$
$$+$$

$$b/(r+\sigma) \qquad\qquad\qquad\qquad + \atop - \quad \dot{\alpha} = 0$$

$$\tilde{R}_0 K \qquad y(t)$$