UNIVERSITY
OF OSLO
HEALTH ECONOMICS
RESEARCH PROGRAMME

Individual and household value of mortality reductions with intrahousehold bargaining

Jon Strand
Department of Economics,
The Ragnar Frisch Centre for Economic Research and HERO

Working Paper 2004: 2

Individual and household value of mortality reductions with intrahousehold bargaining

Jon Strand*

Health Economics Research programme at the University of Oslo
HERO 2004

Keywords: Value of statistical life; household bargaining; intertemporal allocation models; optimal life insurance.

* Author's address: Department of Economics, University of Oslo, P.O. Box 1095 Blindern N-0317 Oslo (fax:(47)22855035)
e-mail: **jon.strand@econ.uio.no**

Health Economics Research programme at the University of Oslo
Financial support from The Research Council of Norway is acknowledged.
ISSN 1501-9071, ISBN 82-7756-134-2

Abstract

I derive alternative measures of maximum willingness to pay (WTP) and value of statistical life (VSL) related to changes in the supply of a public good affecting mortality for both members of two-person households, when members are selfish, live for at most two periods, and strike efficient Nash bargains over consumption of individual and household goods. I find no systematic bias in letting one household member conduct the (WTP or VSL) valuation on behalf of the household. Publicgood VSL may exceed private-good VSL due to each member attaching (purely selfish) preferences to the event that the other member survives or dies, and to a possible net income potential of the other member when surviving in period 2. When period 2 is a retirement period and household members' incomes are then fixed, interview surveys tend to overvalue VSL due to ignored negative effects of own survival on government pension budgets.

1.Introduction

The literature dealing with the valuation of statistical lives (VSL) typically views mortality risk and risk change as purely private and personal, affecting and defined by one particular individual. Some of this work deals with interpersonal issues by focusing on altruism, i.e., whether and how individuals make concerns for the well-being of others as part of their own expressed valuations. Interpersonal issues become particularly relevant when the objects to be valued are public investments or regulations (such as stricter environmental or public safety policies or general public health improvements) that simultaneously affect the mortality risks of many, perhaps all, individuals in society.[1] A prevailing view is that interpersonal issues here can be disregarded when any possible altruism is of the socalled non-paternalistic type.[2]

In this paper I question this approach and argue that there are several reasons for considering the value others' survival probabilities, even when altruism can be ignored. First, purely selfish preferences may be attached to the survival of others, most importantly ones children and spouse but also other individuals in society. Secondly, the survival of others may affect ones future consumption possibilities, which in turn should affect both private and social valuations. Thirdly, in a household context, it is often far from obvious whether valuation ought to be viewed as conducted individually by one family member (spouse), or jointly by one member valuing the relevant project "on behalf of" the household.

In the model below I consider a family context where two spouses have a common household budget, and allocate their household resources efficiently among two goods, a private (individual) good and a common household good. The household has

[1] See Strand (2003a) for a discussion of relevant issues in this context.

a horizon of two periods. In period 1 both spouses are alive. At the start of period 2 either one or both may die, and death occurrences are independent. Death probabilities are affected by the supply of some public good, Z, out of control of the household. The main questions dealt with in this paper are the following: What value does one household member, and the two together, attach to a marginal increase in Z, either on behalf of oneself only, or on behalf of the entire household, and to what degree do these represent "correct" social values? For simplicity and clarity I focus on a case of purely selfish preferences, where either household member exhibits no altruism toward the other member. This is unrealistic but serves as a useful starting point for subsequent analysis of altruism. Each member is assumed to attach a (negative and purely selfish) utility to the event that the other member dies given that oneself is alive. Members also take into consideration consumption effects resulting from the other spouse surviving or not in the second period. These consumption effects differ according to the bargaining strength of each member within the household (assuming efficient Nash bargaining within the household), the importance of the common household good, and the income potential of each member surviving to period 2.

I study two versions of the model. In section 2 I assume that period 2 consumption levels, for the joint household when both survive and for each household member when only one survives, are given. This is a standard case studied in the literature and may be relevant when insurance, savings and annuities markets are very imperfect, and/or the second period is a retirement period and only fixed public retirement benefits are relevant or allowed. In section 3 I assume the existence of a well-functioning annuities market with complete information about mortality risks, where

[2] For some recent prominent papers exposing such views, see Johansson (1994, 2001a, 2001b).

markets perfectly perceive the risk changes resulting from changes in Z, and first-period annuities contracts are continuously renegotiated.[3]

I focus on three main measures of WTP for a marginal change in Z and for the corresponding implied value of statistical life (VSL, throughout defined in terms of period 2 survival): one individual's private value; the sum of individuals' private values (the overall value); and one individual's value representing the household. In the decision-making context of our model (following Strand (2003b)) the two first values are derived considering one or both individuals' willingness to give up units of the individually private good, while the third value is derived considering one individual's willingness to give up units of the common household good. Section 2 establishes that the last values are generally representative of the household, in the sense that the average of the values over the two members generally coincides with the sum of purely individual values (the second measure). Thus on average, one member represents the entire household correctly. When the two individuals in addition have the same bargaining powers and (marginal and average) survival probabilities, we have a stronger result, that each member, individually, represents the household correctly. This result is independent of altruism, and is due entirely to the (two-good efficient-bargaining) decision structure within the household.

A second main result in section 2 is that public-good (and VSL) valuation involves individual household member concern about the survival of the other member in the absence of altruism, for two separate reasons. The first is a selfish (presumably negative) utility from the event that the spouse dies given that oneself survives. Secondly, survival of the spouse affects ones consumption in the second period given

[3] We thus ignore a third obvious alternative, discussed extensively by Johansson (2001b), namely the case where individuals or households may save (and possibly leave bequests) but where complete annuities markets are not available. Johansson shows that derived VSL measures depend on the

that oneself survives. The first of these factors serves to increase the "correct" measure of VSL beyond that found when only considering individual risk changes. The second factor may increase or reduce expressed VSL, depending on whether presence of the spouse adds to or reduces the value of ones own overall consumption in period 2. With given incomes for each spouse, as assumed in section 2, consumption value is generally higher when the other survives, due to economies of scale in utilizing common-household goods.[4]

The assumption of given state-dependent consumption levels in period 2 is innocuous when period 2 incomes correspond to labor incomes for each person while alive. It is more problematic when they do not, as would e.g. be the case when the second period is interpreted as a retirement period, and households then rely on public pensions which are not (fully) tied to ones own payments into the pension system. Prolonging ones life will then tend burden the public pension fund, which is not properly taken into consideration by individuals valuing changes in Z. In section 3 we correct for this effect by assuming that consumption in each state instead is determined in a competitive private insurance market with optimal state-contingent annuities which are continuously renegotiated so as to leave the insurance provider with zero expected profits as Z changes. We show that (when optimal expected household consumption in period 2 is greater than expected period 2 labor income) this leads to downward corrections for all the three main WTP and VSL measures derived in section 2, and in basically the same way for all three. One main difference occurs when the two members have different bargaining powers. Then the private VSL measure (but not the individual's VSL measure on behalf or the household) is

existence or non-existence of complete annuities markets also when borrowing and/or lending is possible. We leave such extensions to future work.

[4] A presumption throughout is that a widowed person does not remarry in period 2.

adjusted downward in proportion to the individual's relative bargaining power (as this individual is viewed as paying for a larger fraction of the additional insurance cost).

In much of the VSL literature, notably Jones-Lee (1976), Shepard and Zeckhauser (1982), Rosen (1988) and Viscusi (1992), and more recently, Johansson (2001a, 2001b), Johannesson, Johansson and Löfgren (1997), and Bleichrodt and Quiggin (1999), an "individualistic" definition of VSL is used whereby interpersonal issues are deemed irrelevant or unimportant. Exceptions are considerations made for altruism by, among others, Bergstrom (1982), Jones-Lee (1991, 1992), Johansson (1994) and Quiggin (1998). To my knowledge, however, no contribution has considered interpersonal factors in the contexts of concern here.

A debatable issue is what model of household behavior to use as basis for such an analysis. Much of the received (Manser and Brown (1980), McElroy and Horney (1981), Chiappori (1988), Lundberg and Pollack (1993)), and also more recent literature (Browning (2000), Browning and Chiappori (1998), Aura (2002b)) is based on the Nash bargaining model, which has received some recent empirical support (Browning and Chiappori (1998), Lundberg and Ward-Batts (2000), Aura (2002a)). Personally I find the Nash bargaining model theoretically convincing, and preferable to either conflict models or unitary models; see else Bergstrom (1997) for a more general overview.

Note that since preferences under our approach are fully selfish, there will be no inappropriate "double counting" of values even as any one individual attaches value (in the form of WTP) to another person surviving, as has been a concern in some of the cited literature dealing with altruism. This serves to highlight the importance of the extension made here, to incorporate interpersonal factors in deriving appropriate VSL measures.

2. The case of exogenous consumption levels

2.1 The basic model

Consider a household with two individuals (spouses), each of whom lives for at most two periods. In period 1 (the present), both spouses live. In period 2, individual i lives with probability $p_i(Z)$, i = 1,2, where Z denotes the supply of some public good which affects these probabilities. Most reasonably, Z can be identified with the supply of health and environmental goods, provided by the government. We assume that no other goods or activities of the individuals affect p_i (or rather, that the model does not embed any mechanisms by which such goods or activities operate). Assume that the (von Neumann-Morgenstern) intertemporal utility function for individual 1 can be written as

$$(1) \qquad W_1 = u_1(C_{11}) + v_1(H_1) + \delta p_1(Z)\{p_2(Z)[u_1(C_{1B}) + v_1(H_{2B})] + [1-p_2(Z)][u_1(C_{1S}) + v_1(H_{2S}) - L_1]\} .$$

Utilities are assumed to be additively separable across time and types of goods. They depend on a private good C consumed exclusively by each member, and a common household good H. C_i denotes person i's consumption of C in period 1, while C_{iB} and C_{iS} denote person i's private consumption in period 2 when, respectively, both are alive in that period, and only individual i survives to period 2. Correspondingly, H_1 denotes period 1 consumption of H, H_B denotes the common consumption of H in period 2 given that both are alive, while H_{iS} denotes individual i's consumption of H in period 2 given that the spouse is dead. The functions u_i and v_i are both increasing and strictly concave in their arguments, satisfy standard Inada conditions, and are identical for both periods. δ is the discount factor from period 1 to period 2 (assuming for simplicity that the two periods are equally long).[5] The three main terms representing period 2 capture the idea that the consumption level of a given individual (given survival to period 2) is state dependent, and depends on whether or not the spouse also survives. L_i represents a (fixed) utility loss suffered by member i, when the other member dies at the start of period 2 and member i survives.[6] We also assume that for the state where individual i is dead in period 2, this individual attaches no ex ante value to the spouse surviving or dying in that period.[7]

(1) embeds no altruism in the normal sense. Individual 1 attaches preferences to whether or not the spouse is alive in period 2 (and vice versa), but this is not based on altruism but rather because individual 1 attaches a selfish disutility to the event that the spouse dies, and because individual 1's consumption level in period 2 depends on survival of the spouse.

In this version of the model, disposable household income is assumed exogenous in each possible state.[8] Disposable income in period 1 equals R_1, while the disposable income in period 2 equals R_B, R_{12} or R_{22}, depending on whether both survive, only individual 1 survives, or only individual 2 survives.

Assume that the two members, whenever they both live, reach an efficient Nash bargain over their joint consumption of private and common household goods, on the

[5] The utility-theoretic formulation here is quite rudimentary and not very general, e.g., it does not allow for more complex substitution between the individual good, the household good and the public good leading to changes in mortality. In Strand (2003b) I discuss a model which is more complex is these dimensions, and show that the same main points, regarding effects of intra-household bargaining, go through also then.

[6] With our formulation, the effect of the spouse dying is fully captured by the term L_i. More generally, the fuctions u_i and v_i will depend on such an event. Introducing more general such functions however only complicates the analysis without adding much of substance for our purposes here.

[7] In general such values could of course exist. We are here however focusing on the case of purely selfish motivations.

[8] There are thus no savings nor borrowing possibilities between periods. This is restrictive and will be relaxed in section 3 below. In terms of economic reality, and given that period 2 is a retirement period, it is designed to represent a situation where consumption upon retirement is "largely" determined by public retirement benefits that are viewed as fixed by the individuals (but may differ according to whether the individuals survive alone and together).

basis of a common household budget in each of the periods. We first consider the period 1 allocation. The budget constraint for period 1 can be expressed as

$$R_1 = C_1 + C_2 + H_1. \quad (2)$$

The Nash product for the bargain can be written on the form[9]

$$NP_1(1) = \left[u_1(C_1) + v_1(H_1)\right]^{\beta}\left[u_2(C_2) + v_2(H_1)\right]^{1-\beta}. \quad (3)$$

Here β and 1-β represent bargaining strengths of individual 1 and 2. A standard Nash bargaining solution implies that NP(1) is maximized with respect to the C_i and H, subject to the household budget constraint in period 1. We form the Lagrangian

$$F_1(1) = NP(1) - \lambda(C_1 + C_2 + H_1 - R_1), \quad (4)$$

where λ is a Lagrange multiplier associated with the budget constraint. Maximizing (4) with respect to the C_i and H_1 now yields the first-order conditions:

$$\frac{\partial F_1(1)}{\partial C_1} = \beta W_1^{\beta-1} W_2^{1-\beta} u_1^{'}(C_1) - \lambda = 0 \quad (5)$$

$$\frac{\partial F_1(1)}{\partial C_2} = (1-\beta) W_1^{\beta} W_2^{-\beta} u_2^{'}(C_2) - \lambda = 0 \quad (6)$$

[9] We formulate the Nash bargaining solution with threat points of the two spouses both equal to zero. This can be denoted the “inside option” in the sense that it represents the (normalized) utility of each of the two spouses in the case of a breakdown of the negotiations over common resources, but given that marriage is sustained. The spouses could in addition have outside options exceeding zero (possibly representing the option of divorce). Invoking the strategic bargaining literature (e.g. Binmore (1985), Binmore, Rubinstein and Wolinsky(1986), and Muthoo (1999)) these outside options would not influence on the bargaining solution given here, provided that their utility values do not exceed the utilities under the current bargaining solution. This will be assumed here.

(7) $$\frac{\partial F_1(1)}{\partial H_1} = \beta W_1^{\beta-1} W_2^{1-\beta} v_1^{'}(H_1) + (1-\beta) W_1^{\beta} W_2^{-\beta} v_2^{'}(H_1) - \lambda = 0 ,$$

where the W_i are the Nash maximands (expressions inside the respective square brackets in (3)). Eliminating λ permits us to derive the following conditions:

(8)-(9) $$\frac{1}{1+n} u_1^{'}(C_1) = \frac{n}{1+n} u_2^{'}(C_2) = v^{'}(H_1).$$

Using the notation $n = [(1-\beta)/\beta](W_1/W_2)$, n may without much loss of generality be viewed as a "primitive" of the bargaining solution. When $n \to 0$, only member 1 has bargaining power; when n = 1, both members have the same "effective bargaining power" (resulting in particular when utility functions are identical and β = ½); and when $n \to \infty$, only member 2 has bargaining power. We simplify by setting $v_1'(H_j)=v_2'(H_j)=v'(H_j)$, for relevant equilibrium values of H_j, j = 1,2.[10] (8)-(9) give marginal rates of substitution between the respective private goods C_i, and the family good H_1, under efficient Nash bargaining. It is the familiar (Samuelsonean) public-good optimality condition prescribing the marginal value of the household ("public") good to equal the (weighted) sum of marginal values of the private goods.[11] Generally, for $\beta \in (0,1)$ (and thus $n \in (0,\infty)$) $u_i'(C_i)$ will exceed $v_i'(H)$ at an efficient

[10] This is not very restrictive. The utility functions utilized here fulfil standard von Neumann-Morgenstern criteria and are thus invariant to an increasing linear transformation, this transformation may without loss of generality be chosen to equalize absolute and marginal utilities of common household consumption at this point, in either of the periods. If the H_j level then does not change much between the two periods, our assumption will hold approximately for any valid utility function specification.

[11] See e.g. Starrett (1988). Our solution here of course also coincides with the analysis of Coase (1960), and our model assumes that the "Coase theorem" holds for intrafamily allocations. We will claim that if the Coase theorem is to hold approximately anywhere, it is likely to hold for intrafamily allocations where the setting is explicitly cooperative and individual interact almost continuously. For other presentations of efficient intrafamily bargaining, although without explicit consideration for common

Nash bargaining solution. This can be understood by considering the effects on member 1's utility, when R increases by one (small) unit. This unit can be used either to increase consumption of the common good H_1, or of private goods C_i. In the latter case household member 1 only receives a fraction 1/(1+n) of income to be spent on increased personal consumption. The consumption value of this increased personal consumption must in optimum equal the consumption value of the unit increase in H, which in turn implies that the marginal utility of additional personal consumption must be higher than that of common consumption.

As β tends to one (n tends to zero), the solution becomes "dictatorial" as person 1 alone decides on the common budget. Then (with no altruism) only person 1 enjoys private consumption in the limit, and the rate of substitution between C_1 and H_1 is unity.[12] (Conversely, as β tends to zero and n to infinity, only person 2 enjoys personal consumption and the rate of substitution between C_2 and H_1 tends to unity.)

When both spouses live to period 2, the solution is then still characterized by (8)-(9), with the only difference that C_i and H_1 are replaced by C_{iB} and H_B. When only one spouse lives the conditions are instead

$$(10) \qquad u_1^{'}(C_{iS}) = v^{'}(H_{iS}), i = 1,2.$$

Then there is of course no bargaining in period 2. Note that the rule (10) for a singly surviving individual i in period 2, is the same as the rule for this individual, given that he or she has bargaining strength equal to unity and the other person surviving.

household goods, see Manser and Brown (1980), McElroy and Horney (1981) and Lundberg and Pollack (1993).

2.2 WTP and VSL measures of changes in Z in the basic model

We now consider ways in which one family member, and the two members together, attach value in the form of willingness to pay (WTP), to a (small) change in the supply of the public good Z, that affects the survival probability of both household members in the second period. We will also derive value of statistical life (VSL) measures that are related to the respective WTP measures.

The related WTP concepts differ in the following two dimensions:

a) as each individual's WTP for the resulting change in his or her own death probability only; or alternatively, for the resulting simultaneous change in the death probabilities of both members.

b) as each person's purely private WTP (on behalf or himself or herself only) for the relevant death probability changes; or alternatively, as person 1' WTP on behalf of the entire household.

Dimension a) distinguishes between private and household mortality changes, and dimension b) between private and household WTP. With regard to dimension a), most of the current VSL literature treats mortality reduction as a private good, as noted in section 1 above. This is often justified with reference to the idea that altruism can be ignored or made irrelevant for proper VSL measure.[13] I here abstract from altruism and instead focus on other possible reasons why the private- versus public-good issue is still relevant, namely selfish preferences for and income effects of other individuals' survival. Dimension b) is less discussed in the literature. In many

---

[12] Note that for limit solutions of $C_{i1} = 0$ to make sense, we must allow for the household good H to contain all elements necessary for basic survival, such as basic food consumption and clothing.

[13] Most theoretical and empirical treatments of the value of statistical life (VSL) issue depart from such an assumption; see e.g. the theoretical analyses by Shepard and Zeckhauser (1982), Rosen (1988), Jones-Lee (1992), Quiggin (1998) and Johansson (1994, 2001a), and the empirical analyses by Johannesson et.al. (1997), Alberini et.al. (2002a, 2002b). Johansson (1994) in particular notes that when altruism is nonpaternalistic and other individuals' utilities are assumed to be kept constant during the valuation procedure, any altruistically motivated risk changes for other individuals will not affect

empirical WTP studies formulations such as "what is your purely personal WTP" and "what is your WTP on behalf of the household" are typically used interchangeably, without much discussion. More recently Strand (2002) has addressed this issue in the context of a related model where a general public good is valued (and mortality is not an issue) under altruism. Here altruism plays no role, but otherwise the basic idea is similar except that the public good considered only affects mortality and the problem by its nature is intertemporal.

I simplify (1) as follows:

$$W_1 = u_1(C_1) + v_1(H_1) + \delta p_1(Z)\{p_2(Z)V_1(B2) + [1-p_2(Z)][V_1(S2) - L_1]\} - \delta(1-p_1)L_1. \tag{1a}$$

$V_1(B2)$ and $V_1(S2)$ represent utilities of consumption for person 1 in period 2, when both household members are alive, and only person 1 is alive, respectively.

We define 4 concepts of WTP for changes in mortality risks associated with a marginal change in Z, labelled A-D in the following.[14] Note again that the R's are held constant; this assumption will be relaxed in the next section.

A. Purely private value of own mortality risk reduction only (MA). We first consider WTP in terms of a reduced amount of the private good for person 1, which keeps this person's total utility constant, given a marginal change in Z which is considered to affect only p_1. We consequently hold p_2 constant. This is hypothetical and serves mainly as a reference for the other three, more realistic, cases. The measure may still

the correctly expressed WTP measure. For an empirical analysis that explicitly considers the public-good aspects of VSL see Strand (2002b).

[14] All expressions in cases A-D are derived for person 1. Similar expressions for person 2 can be found exchanging footscripts where necessary.

be relevant under the following question framing, standard in contingent valuation (CV) surveys: "What is your personal maximum willingness to pay for a marginal increase in the public good Z, when considering only the effect on your own survival probability?" Denoting the ex ante expected utility in period 2 conditional on surviving for individual 1 by $EV_1(2)$, we have

(11) $$u_1'(C_1)dC_1 + \delta EV_1(2)p_1'(Z)dZ = 0$$

which implies

(12) $$MA(1) = -\frac{dC_1}{dZ}(p_2 = const.) = \frac{\delta EV_1(2)p_1'(Z)}{u_1'(C_1)}\,.$$

From Rosen (1988), the implicit measure of the value of statistical life (VSL), here taken to mean the saving of one statistical life in period 2 as viewed from period 1 using measure A, can be defined as follows:[15]

(13) $$V(MA1) = -dC_1/dp_1 = MA(1)/p_1'(Z)\,.$$

Absent any concern for others than the individual conducting the valuation in this case, VSL can be defined simply as the marginal willingness to pay for an increase in survival probability, for the one individual affected.

[15] Rosen defines VSL as the rate of substitution between wealth (consumption) and risk for one individual, which corresponds to our definition in (13).

B. Purely private value of overall family mortality risk reduction. Here I also include (more reasonably) person 1's valuation of the impact on person 2's risk of dying when Z changes. This leads to the expression

(14) $$u_1'(C_1)dC_1 + \delta\left[EV_1(2)p_1' + p_1(V_1(B2) - V_1(S2) + L_1)\right]dZ = 0$$

This yields the following marginal private WTP for mortality risk reduction as a public good:

(15) $$MB(1) = -\frac{dC_1}{dZ} = \frac{\delta\left[EV_1(2)p_1'(Z) + p_1(Z)p_2'(Z)(V_1(B2) - V_1(S2) + L_{12})\right]}{u_1'(C_1)}.$$

MB(1) may be higher of lower than MA(1), depending on whether the expression

(16) $$\Delta V_{12} = V_1(B2) - V_1(S2) + L_1$$

is positive or negative. ΔV_{12} expresses the utility loss for member 1 in period 2, as a result of the other member dying at the start of that period. Reasonably, $L_1 \geq 0$ (individual 1 does not want his or her spouse dead for its own sake). ΔV_{12} is then always positive when the utility of period 2 consumption for person 1 surviving, is greater when also the other spouse survives than when only person 1 survives. This expression will tend to be positive when person 1 has higher overall consumption when joined with his or her spouse than when living alone. It may perhaps be argued that $V_1(B2) - V_1(S2) \geq 0$ is the typical case, and that thus typically $\Delta V_{12} > 0$.[16]

The corresponding VSL expression, V(MB1), may now be defined by the sum of values for the two individuals, associated with survival of individual 1.[17] This leads to the following expression:

(17) $$V(MB1) = \frac{MA(1) + MB(2) - MA(2)}{p_1'(Z)}.$$

Here $MA(1)/p_1'(Z) = V(MA1)$, from (13). From (17), $MB(2)-MA(2) > 0$ when $\Delta V_{21} > 0$, which, as argued above, is typically the case. From (8)-(9), (12) and (15),

(17a) $$V(MB1) = \frac{\delta}{(1+n)v'}\left[EV_1(2) + np_2\Delta V_{21}\right]$$

(17b) $$V(MB2) = \frac{\delta}{(1+n)v'}\left[n(EV_2(2) + p_1\Delta V_{12}\right].$$

The last terms in (17a)-(17b) are proportional to p_2 and p_1 respectively. Intuitively, in (17a), person 2's WTP for increased survival probability of person 1 depends on person 2's ability to benefit from person 1's presence in period 2, which occurs only when person 2 survives.

[16] It is of course possible that $V_1(B2) - V_1(S2) < 0$ in some cases, in particular when the household in period 2 lives mainly out of commonly saved assets.
[17] This implies a simple generalization of Rosen's (1988) VSL definition, as the sum of individuals' marginal rates of substitution between "wealth" and risk, when risk changes for one given individual.

C. Individual value of overall family risk reduction, on behalf of the household. We now elicit person 1's marginal WTP on behalf of the household (and thus in terms of the common household good H), for the common change in probabilities just considered. This measure is defined by

(18) $$MC(1) = -\frac{dH_1}{dZ} = \frac{\delta\left[EV_1(2)p_1'(Z) + p_1(Z)p_2'(Z)\Delta V_{12}\right]}{v'(H_1)},$$

with an equivalent expression for person 2. From (8)-(9), MC(1) = (1+n)MB(1) It is here less obvious than in case B, how VSL should defined. What is valued here is the joint risk change for both individuals. One possible VSL measure is based on one member's valuation of this joint risk change. This measure is for members 1 and 2 respectively,

(19) $$V(MC1) = \frac{1+n}{2}\left[\frac{MA(1)}{p_1'(Z)} + \frac{MB(1) - MA(1)}{p_2'(Z)}\right]$$

(19a) $$V(MC2) = \frac{1+n}{2n}\left[\frac{MA(2)}{p_2'(Z)} + \frac{MB(2) - MA(2)}{p_1'(Z)}\right].$$

One must here divide by 2 since two statistical lives are simultaneously valued. Also here we find alternative expressions for V(MCi), using (8)-(9), (12) and (15),

(19b) $$V(MC1) = \frac{\delta}{2v'(H_1)}\left[EV_1(2) + p_1\Delta V_{12}\right]$$

(19c) $$V(MC2) = \frac{\delta}{2v'(H_1)}\left[EV_2(2) + p_2\Delta V_{21}\right].$$

Another possible VSL measure for the household is based on valuation expressed by both individuals (and in each case "representing the entire household"), possibly the average of the two. This leads to the following measure:

(20) $$V(MC3) = \frac{1+n}{4}\left[\frac{MA(1)}{p_1'(Z)} + \frac{MB(1) - MA(1)}{p_2'(Z)}\right] + \frac{1+n}{4n}\left[\frac{MA(2)}{p_2'(Z)} + \frac{MB(2) - MA(2)}{p_1'(Z)}\right].$$

V(MC3) is constructed simply as an arithmetic average of the two individual measures V(MC1) and V(MC2). An alternative way of expressing V(MC3) is

(20a) $$V(MC3) = \frac{\delta}{4v'(H_1)}\left[EV_1(2) + p_1\Delta V_{12} + EV_2(2) + p_2\Delta V_{21}\right].$$

D. Overall household value of the total risk reduction when Z changes. I finally derive measures of the household's overall value of a marginal change in Z, as the sum of purely private WTP for the overall changes in risk. Adding up MB(1) and MB(2) yields

(21) $$MD = \frac{\delta}{(1+n)v'(H_1)}\begin{bmatrix}(p_1'(Z)EV_1(2) + p_1(Z)p_2'(Z)\Delta V_{12} \\ + n(p_2'(Z)EV_2(2) + p_2(Z)p_1'(Z)\Delta V_{21}\end{bmatrix}.$$

MD is a weighted average of MC(1) and MC(2) with weights 1/(1+n) and n/(1+n). Arguably, MD is the appropriate measure of aggregate household WTP for a given change in Z, while MC(1) and MC(2) are more reasonable measures of individual

WTP on behalf of the household for the same change in Z. There is here no systematic bias in, say, the measure MC(1) when taken to represent MD. That this happens in the complete absence of altruism is a novel result. It depends on the existence of a common household good together with an assumption of efficient intrahousehold bargaining, over personal and household-level consumption.

Also for MD, it is straightforward to define an associated measure of average VSL, as the arithmetic average of V(MB1) and V(MB2):

$$(22) \qquad V(MD) = \frac{1}{2}\left[\frac{MA(1) + MB(2) - MA(2)}{p_1'(Z)} + \frac{MA(2) + MB(1) - MA(1)}{p_2'(Z)}\right].$$

V(MD) is a simple average of the individual VSL values, whereby in each case total individual valuation is evaluated at the individual marginal probability hazard. Alternatively we may write

$$(22a) \qquad V(MD) = \frac{\delta}{2(1+n)v'(H_1)}\left[EV_1(2) + p_1\Delta V_{12} + n(EV_2(2) + p_2\Delta V_{21})\right].$$

(22a) is identical to (20a) (the average VSL measure over the two individuals “on behalf of the household”) except that weighting of the two individuals’ valuations are different. The two coincide for the special case of n = 1 (symmetric bargaining power) and $p_1' = p_2'$ (equal death risk). Perhaps equally interesting is the comparison between (22), and the related individual measure (for member 1) on behalf of the household, in (19). These coincide when n = 1 and $p_1' = p_2'$, and in addition MB(1) = MB(2). This is a symmetric case where members' preferences, bargaining powers and death probabilities are all the same. In other cases V(MD) differs from V(MCi). Typically, the latter exceeds the former when $n > 1$ (member 2 has the greater bargaining power) and when the square bracket in (19b) is greater than that in (19a) (member 2 has the greater utility value of surviving).

3. The case of complete annuities markets

3.1 The model

In this section we assume that consumption expenditures in each state and period are no longer exogenous but instead chosen optimally by the household members, jointly in period 1. The two members still bargain over a joint utility surplus, but now with optimal saving or dissaving between periods, together with the respective consumption choices, and leaving no bequest. We assume that the household has two types of assets available for present and future consumption. First, (exogenous) labor income A_{ij}, for member i in period j (= 1,2), as long as the respective member survives, and secondly, a non-labor asset R_1 available to the household at the start of period 1. The household can now borrow or lend freely at a given interest rate corresponding to the discount factor δ. It can also freely contract with insurers, about payments in period 2 contingent on member 1 and/or 2 surviving to that state. We assume the existence of a competitive annuities market where future insurance payments are actuarially fair, and with symmetric information. It implies that when p_1 and/or p_2 change (as the supply of the public good Z changes), the insurance contract is modified so as to achieve actuarial fairness.

The household's budget constraint can in this case be written as

$$ER_0 \equiv A_{11} + A_{21} + R_1 + \delta\left[p_1(Z)A_{12} + p_2(Z)A_{22}\right] = ER$$

(23)

$$\equiv C_1 + C_2 + H_1 + \delta\begin{bmatrix} p_1 p_2 (C_{1B} + C_{2B} + H_B) \\ + p_1(1-p_2)(C_{1S} + H_{1S}) + (1-p_1)p_2(C_{2S} + H_{2S}) \end{bmatrix}.$$

Here ER_0 is interpreted as the fixed initial household wealth, for the initial level of Z. Expected utility for person 1 is now still given by (1), and expected utility for 2 by a corresponding expression shifting subscripts where appropriate. We specify the expected utilities of individuals 1 and 2 as follows:

(24)
$$W_1 = u_1(C_1) + v(H_1) + \delta p_1(Z)\{p_2(Z)[u_1(C_{1B}) + v_1(H_B)] + [1-p_2(Z)][u_1(C_{1S}) + v(H_{2S}) - L_1]\}$$

(25)
$$W_2 = u_2(C_2) + v(H_2) + \delta p_2(Z)\{p_2(Z)[u_2(C_{2B}) + v_2(H_B)] + [1-p_1(Z)][u_2(C_{2S}) + v(H_{2S}) - L_2]\}.$$

We form the Lagrangean

(26)
$$F_2 = (W_1)^{\beta}(W_2)^{1-\beta} - \lambda(ER - ER_0).$$

Maximizing (26) with respect to C_1, C_2, H_1, C_{1B}, C_{2B}, H_B, C_{1S}, C_{2S}, H_{1S} and H_{2S} now yields a set of first-order conditions given in the appendix. The following conditions are fulfilled under an optimal intertemporal household allocation:

(27)-(28)
$$C_i = C_{iB} = C_{iS}, \ i = 1,2$$

(29)-(30) $$\frac{1}{1+n}u_1^{'}(C_1) = \frac{n}{1+n}u_2^{'}(C_2) = v^{'}(H_1)$$

(31)-(32) $$\frac{1}{1+n}u_1^{'}(C_{1B}) = \frac{n}{1+n}u_2^{'}(C_{2B}) = v^{'}(H_B)$$

(33) $$u_1^{'}(C_{iS}) = v^{'}(H_{iS}), i = 1,2 .$$

In the optimal household allocation, private consumption is now simply constant across states for a given individual.[18] (8)-(9) hold for the optimal allocation of the common household good, as before in period 1 (as in the solution with given period 2 incomes above), and now also when both household members are alive in period 2. Finally, when each of the two individuals is single in period 2, the marginal utilities of the private and the household good are equal for a given surviving individual.

These solutions have implications for $V_i(B2)$ and $V_i(S2)$ defined in section 2, and for interpreting the value measures MB, MD and MF. We find the following relationship:

(34) $$V_i(B2) – V_i(S2) + L_i = V(H_B)-V(H_{iS}) + L_i, \quad i = 1,2.$$

3.2 Valuation of risk changes under complete annuities markets

Consider now a change in Z that changes p_1 and p_2, and assume that (23) holds, i.e. the household is offered actuarially fair insurance in period 2, under symmetric

[18] This is of course a general property of an optimal intertemporal allocation in this case, which is here achieved.

information on probabilities and no moral hazard.[19] Consider then first the case where consumption in each state in period 2 is kept constant, and H_1 changes to fulfil the budget constraint (23) (expressing actuarial fairness). We find (letting primes denote derivatives with respect to Z, and letting C_i denote the optimal and fixed level of consumption for member i)

(35)
$$\begin{aligned}&\delta p_1'\left[A_{12} - C_1 - p_2 H_B - (1-p_2)H_{1S} + p_2 H_{2S}\right]dZ\\&+\delta p_2'\left[A_{22} - C_2 - p_1 H_B - (1-p_1)H_{2S} + p_1 H_{1S}\right]dZ \equiv \delta \Delta A dZ = dH_1\end{aligned}$$

Here ΔA expresses the net increase in revenue, when Z increases marginally, for an insurance company offering actuarially fair life insurance. When $\Delta A < 0$, the insurance company would lose revenue on the initial contract in period 2 when Z increases and the insurees' expected lifetimes are prolonged. This happens when the A_{i2} are small relative to the period 2 consumption levels, as would be the case when period 2 is interpreted as a retirement period. This revenue loss must in a competitive market equilibrium for the insurance company be compensated with lower net benefits to be paid out in the surviving states in period 2, or higher net payments by the household to the insurance company in period 1.

When discussing values of increased Z, the (hypothetical) case A, with purely private risk reductions, now makes little sense as the household by force of logic must consider the budget effect of the simultaneous increase in life expectancy for both members when Z changes. Thus cases B and C are relevant in terms of one member

[19] The requirement that budget balance hold for the insurance company at each time and for each possible chosen value of Z requires formally that insurance contracts need to be continuously renegotiated as Z changes. We will not go into the realism of this in practice, only notice that if they are not renegotiated, and the government (after initial annuities contracts have been established) chooses to increase Z in "surprise" fashion, there may be a windfall gain to households that is counterbalanced by a loss to insurance companies.

(1) conducting the valuation. In case C, where member 1 conducts the valuation on behalf of the household, MC(1) is still valid except that it must be modified by the term $\delta\Delta A$. Denoting member 1's valuation of the change in Z on behalf of the household in this case by AC(1), we simply have

(36) $$AC(1) = MC(1) + \delta\, \Delta A.$$

To derive the related VSL measure, note that when member 1 conducts the valuation on behalf of the household, since preferences are purely selfish, member 1 ignores person 2's benefit (or loss) related to the last term in (36) (which is a loss directly in terms of members' own consumption). This implies that the VSL measures in this case, corresponding to the measures V(MC1) and V(MC2) in (19)-(20), can be written simply as

(37)-(38) $$V(ACi) = V(MCi) + \frac{\delta\Delta A}{2p_i'(Z)}, i = 1,2.$$

We must here divide by 2 in the last term since individual i in effect values 2 statistical lives.

Consider next the purely private WTP of member 1 associated with a change in Z. We must then consider the bargaining relationship between the spouses, which implies that member 1 is required to pay a fraction 1/(1+n), and member 2 a fraction n/(1+n), of a reduction in their common period 1 budget for private consumption. Thus member 1 will experience a (likely negative) change in private period 1 consumption equal to δ ΔA/(1+n). Consequently, the purely private WTP for members 1 and 2 can be expressed as

(39) $$AB(1) = MB(1) + \frac{\delta\Delta A}{1+n}$$

(39a) $$AB(2) = MB(2) + \frac{n\delta\Delta A}{1+n}.$$

The corresponding VSL measures are here, in similar fashion as for the AC1i,

(40) $$V(AB1) = V(MB1) + \frac{\delta\Delta A}{(1+n)p_1'(Z)} \; .$$

(40a) $$V(AB2) = V(MB2) + \frac{n\delta\Delta A}{(1+n)p_2'(Z)} \; .$$

Finally, we may construct an average VSL measure for private values, corresponding to V((MD), as follows:

(41) $$V(AD) = \frac{1}{2}\left[V(MB1) + \frac{\delta\Delta A}{(1+n)p_1'(Z)} + V(MB2) + \frac{n\delta\Delta A}{(1+n)p_2'(Z)}\right].$$

In all these cases VSL must be adjusted by the consumption change resulting from the change in the fair annuities contract when life spans increase due to increased Z. This correction is done in "essentially" the same way in all three cases considered (cases B-D in section 2 above). The main difference between the corrections stems from the bargaining parameter n. When considering private values, in (40), (40a) and (41), a low n implies that member 1 has high bargaining power, implying that he or she pays a large share of the extra cost $\delta\Delta A$ when the annuities contract is revised.

4. Implications and final comments

There are three main implications of this analysis, for public-good valuation and VSL derivation in particular, which are new in the literature. First, eliciting VSL from one individual in a two-person household, under efficient intra-household bargaining over the allocation of personal and household goods, implies that the individual on average represents the household correctly. Secondly, there are reasons apart from altruism for considering VSL as a (local) public good, and not solely as a private good which is more customary in the literature. The reasons are that individuals may be (selfishly) concerned for, and their future consumption possibilities may depend on, the survival of others (primarily close family members). I argue in section 2 that such factors typically lead to increased VSL as a public good, beyond its pure private-good value. Thirdly, when VSL measures are derived assuming that changes in future individual consumption levels do not fully reflect changes in future individual work incomes as life spans are extended (as e.g. is typically the case under fixed or pay-as-you-go public pension schemes), these measures are biased upward, and more so the older are the persons surveyed, as older individuals tend to have lower future work incomes. This factor may serve to explain the (for some researchers puzzling) finding, that VSL as derived from interview surveys are found to drop only very little with the interviewee's age.[20]

[20] See e.g. Alberini et. al. (2002a,b). Our analysis may help to bridge some of the conceptual gap between this extreme individual-based welfare economics view of VSL, and the (equally extreme but opposite) health-profession view, represented by the QALY concept whereby VSL is considered as

Appendix: Optimal household allocation with perfectly competitive annuities markets

This problem is solved maximizing the expression (26) with respect to, in order, C_1, C_2, H_1, C_{1B}, C_{2B}, C_{1S}, C_{2S}, H_B, H_{1S} and H_{2S}, under constraints (23)-(25), and yields the following 10 first-order conditions:

(A1) $$\beta W_1^{\beta-1} W_2^{1-\beta} u_{11} - \lambda = 0$$

(A2) $$(1-\beta) W_1^{\beta} W_2^{-\beta} u_{21} - \lambda = 0$$

(A3) $$\beta W_1^{\beta-1} W_2^{1-\beta} v'(H_1) + (1-\beta) W_1^{\beta} W_2^{-\beta} v'(H_1) - \lambda = 0$$

(A4) $$\delta p_1 p_2 \left[\beta W_1^{\beta-1} W_2^{1-\beta} u_{1B} - \lambda\right] = 0$$

(A5) $$\delta p_1 p_2 \left[(1-\beta) W_1^{\beta} W_2^{-\beta} u_{2B} - \lambda\right] = 0$$

(A6) $$\delta p_1 (1-p_2) \left[\beta W_1^{\beta-1} W_2^{1-\beta} u_{1S} - \lambda\right] = 0$$

(A7) $$\delta (1-p_1) p_2 \left[(1-\beta) W_1^{\beta} W_2^{-\beta} u_{2S} - \lambda\right] = 0$$

(A8) $$\delta p_1 p_2 \left[\beta W_1^{\beta-1} W_2^{1-\beta} v'(H_B) + (1-\beta) W_1^{\beta} W_2^{-\beta} v'(H_B) - \lambda\right] = 0$$

---

proportional to expected remaining lifetime; see Hammitt (2002) for a further analysis and comparison. Note however that some recent studies indicate the possibility that VSL need not necessarily drop with age. Sun and Ng (2002) argue that VSL may increase with age, even when the value of time is higher

(A9) $$\delta p_1(1-p_2)\left[\beta W_1^{\beta-1}W_2^{1-\beta}v'(H_{1S})-\lambda\right]=0$$

(A10) $$\delta(1-p_1)p_2\left[(1-\beta)W_1^{\beta}W_2^{-\beta}v'(H_{2S})-\lambda\right]=0$$

From these equations we derive conditions (8)-(9) for C_1, C_2 and H_1, and identical conditions for C_{1B}, C_{2B} and H_B. In addition we find that the condition for C_{1S} is identical to the condition for C_1, while the condition for C_{2S} is identical to the condition for C_2. We also find that

(A11) $$u_{1S} = v'(H_{1S}),\ u_{2S} = v'(H_{2S}).$$

for the young. Johansson (2001b) points out that VSL is likely to be higher in mid-age, and possibly higher for the old than for the very young. See also Johansson (2003) for a general discussion.

References

Alberini, A., Krupnick, A., Cropper, M. , Simon, N. and Cook, J. (2002a), The willingness to pay for mortality risk reduction: A comparison of the United States and Canada. CESifo working paper 668.

Alberini, A., Krupnick, A., Cropper, M. , Simon, N.,O'Brien, B., Goeree, R. and Heintzelman, M. (2002b), Age, health and the willingness to pay for mortality risk reductions: A contingent valuation survey of Ontario residents. Journal of Risk and Uncertainty.

Aura, S. (2002a), Does the balance of power within a family matter? The case of the Retirement Equity Act. CESifo working paper no. 734.

Aura, S. (2002b), Uncommitted couples: Some efficiency and policy implications of marital bargaining. CESifo working paper no. 801.

Bergstrom, T. C. (1982), When is a man's life worth more than his human capital?. In M. W. Jones-Lee (ed.): The value of life and safety: Proceedings of a conference held by the Geneva Association, pp 3-26. Amsterdam: North-Holland.

Bergstrom, T. C. (1997), A survey of theories of the family. In M. Rosezweig and O. Stark (eds.): Handbook of population and family economics, vol. 1A, pp. 21-79. Amsterdam: North-Holland.

Binmore, K. (1985), Bargaining and coalitions. In A. Roth (ed.): Game-theoretic models of bargaining. Cambridge: Cambride University Press.

Binmore, K., Rubinstein, A. and Wolinsky, A. (1986), The Nash bargaining solution in economic modeling. Rand Journal of Economics, 17, 176-188.

Bleichrodt, H. and Quiggin, J. (1999), Life-cycle preferences over consumption and health: when is cost-effectiveness analysis equivalent to cost-benefit analysis? Journal of Health Economics, 18, 681-708.

Browning, M. (2000), The saving behaviour of a two-person household. Scandinavian Journal of Economics, 102, 235-251.

Browning, M. and Chiappori, P. A. (1998), Efficient intra-household allocations: A general characterization and empirical test. Econometrica, 66, 1241-1278.

Chiappori, P. A. (1988), Rational household labor supply. Econometrica, 56, 63-90.

Coase, R. (1960), The problem of social cost. Journal of Law and Economics, 3, 1-44.

Hammitt, J. D. (2002), QALYs versus WTP. Risk Analysis, 22, 985-1001.

Johannesson, M., Johansson, P. O. and Löfgren, K. G. (1997), On the value of changes in life expectancy: Blips versus parametric changes. Journal of Risk and Uncertainty, 15, 221-239.

Johansson, P. O. (1994), Altruism and the value of statistical life: empirical implications. Journal of Health Economics, 13, 111-118.

Johansson, P. O. (2001a), Is there a meaningful definition of the value of a statistical life? Journal of Health Economics, 20, 131-139.

Johansson, P. O. (2001b), On the definition of the value of a statistical life: A review. Paper presented at the Oslo Workshop on Health Economics, June 2001.

Johansson, P. O. (2003), The value of a statistical life: theoretical and empirical evidence. Applied Health Economics and Health Policy 2003, special issue, 25-33.

Jones-Lee, M. W. (1976), The value of life: An economic analysis. London: Martin Robertson.

Jones-Lee, M. W. (1991), Altruism and the value of other people's safety. Journal of Risk and Uncertainty, 4, 213-219.

Jones-Lee, M. W. (1992), Paternalistic altruism and the value of statistical life. Economic Journal, 102, 70-90.

Lundberg, S. and Pollak, R. (1993), Separate spheres bargaining and the marriage market. Journal of Political Economy, 101, 988-1011.

Lundberg, S. and Ward-Batts, J. (2002), Saving for retirement: Household bargaining and household net worth. Forthcoming, Journal of Public Economics.

Manser, M. and Brown, M. (1980), Marriage and household decision theory – a bargaining analysis. International Economic Review, 21, 21-34.

McElroy, M. and Horney, M. (1981), Nash-bargained decisions: toward a generalization of the theory demand. International Economic Review, 22, 333-349.

Muthoo, A. (1999), Bargaining theory with applications. Cambridge: Cambridge University Press.

Quiggin, J. (1998), Individual and household willingness to pay for public goods. American Journal of Agricultural Economics, 80, 58-63.

Rosen, S. (1988), The value of changes in life expectancy. Journal of Risk and Uncertainty, 1, 285-304.

Shephard, D. S. and Zeckhauser, R. J. (1982), Life-cycle consumption and willingness to pay for increased survival. In M. W. Jones-Lee (ed.): Valuation of life and safety. Amsterdam: North-Holland.

Starrett, D. A. (1988), Foundations of public economics. Cambridge: Cambridge University Press.

Strand, J. (2002), Public- and private-good values of statistical lives: Results from a combined choice-experiment and contingent-valuation survey. HERO working paper, 2002:2, University of Oslo.

Strand, J. (2003a), Interpersonal issues in the valuation of statistical lives. Working paper, Department of Economics, University of Oslo.

Strand, J. (2003b), Public-good valuation and intrafamily allocation. HERO working paper, 2003:20, University of Oslo.

Sun, G. Z. and Ng, T. K. (2002), Time is more valuable for the young, life is more valuable for the old. Working paper, Monash University.

Viscusi, W. K. (1992), Fatal tradeoffs. Public and private responsibilities for risk. New York: Oxford University Press.