

**EXAM IN HMET4100, 25 JANUARY 2019:
SUGGESTED SOLUTIONS AND COMMENTS**

VERSION INTENDED FOR CANDIDATES

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INTRODUCTION

The exam was arranged digitally on the Inspira platform and consisted of 9 questions. Three of the questions were multiple choice questions where the candidates should choose one out of a set of alternative answers. The remaining questions were in "essay" format and the answers had to be written as text.

The multiple choice questions were assessed automatically, while the "essay" questions were assessed manually. The maximum score for each question was 10 points and the minimum was 0 points. In total, the maximum score was 90 points.

The following relation between grades (A-F) and points [0-90] was used: A: [81-90], B: [71-80], C: [55-70], D: [41-54], E: [30-40], F: [0-29].

Explanation of grades. After the grades are published, the candidates will have access to their individual scores by logging on to Inspira. The present suggested solution, which will be distributed to the candidates, explains the general principles for how scores and grades were awarded. A candidate's detailed scores on the exam questions and the general suggested solution are together considered as the formal explanation of the grade, and no further formal explanations will be given.

Appeal against grades. Appeals against grades must be submitted following the procedure described on the course web-page (that is, not submitted to the course coordinator).

Informal feedback. Candidates who wish to receive additional feedback can ask the course coordinator for an informal talk (phone call or personal meeting). The candidates must then email the course coordinator 2-3 suggested times. These suggested times should be within normal office hours (0900h-1700h), with at least 48 hours notice, and within 22 February 2019.

SUGGESTED SOLUTIONS

QUESTION 1

$$\begin{aligned} P(6 < X < 18) &= P(X = 7) + P(X = 8) + \dots + P(X = 17) \\ &= P(X \leq 17) - P(X \leq 6) = 0.9444 - 0.0512 \approx \underline{\underline{0.893}}. \end{aligned}$$

QUESTION 2

Let $p = 0.08$ denote the assumed population proportion. If we let this value replace \hat{p} in the formula for the confidence interval we find

$$n = \frac{p(1-p)(Z_{\alpha/2})^2}{ME^2} = \frac{0.08 \cdot 0.92 \cdot 2.576^2}{0.02^2} = 4883.9,$$

that is, 4884 when rounding up.

QUESTION 3

The outcome X of a sign test follows a binomial distribution, in this case with $n = 11$ trials and with $p = 0.5$ as the probability of success in each trial. The probability of observing 11 successes is then $P(X = 11) = 0.0005$. Thus, when the alternative hypothesis is two-sided, the p-value equals $2 \cdot P(X = 11) = \underline{\underline{0.0010}}$.

QUESTION 4

The test statistic has a standard normal distribution, and the critical value is $Z_c = 1.645$. Assuming H_0 is true, the corresponding critical value for the sample mean is

$$\bar{X}_c = Z_c \cdot \frac{\sigma}{\sqrt{n}} + \mu = 1.645 \cdot \frac{\sqrt{25}}{\sqrt{121}} + 100 = 100.748.$$

The probability of type II error is

$$\begin{aligned} P(\bar{X} < \bar{X}_c | \mu = 102) &= P\left(\frac{\bar{X} - 102}{\sigma/\sqrt{n}} < \frac{\bar{X}_c - 102}{\sigma/\sqrt{n}}\right) \\ &= P\left(Z < \frac{100.748 - 102}{5/11}\right) = P(Z < -2.76) = \underline{\underline{0.0029}}. \end{aligned}$$

QUESTION 5

The critical value is $F_{K-1, n-K, \alpha} = F_{2, 27, 0.05} = 3.354$, the test statistic is

$$F_{obs} = \frac{MSG}{MSW} = \frac{42.2230525}{16.6671037} = \underline{\underline{2.553}},$$

thus, we keep the null hypothesis.

QUESTION 6

Inserting values into the formula, $\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$, we obtain $2.057 \pm 1.96 \frac{1.440773}{\sqrt{1000}}$, which equals (1.97, 2.15).

QUESTION 7

Using the normal approximation, we first obtain

$$E(T) = \frac{n(n+1)}{4} = 162.5,$$

$$Var(T) = \frac{n(n+1)(2n+1)}{24} = 33150,$$

and observe that $T = \min(442, 183) = 183$. The observed test statistic is then

$$Z = \frac{183 - 162.5}{\sqrt{33150}} = 0.113.$$

If we have a two-sided alternative hypothesis, the p-value is $2 \cdot P(Z < -0.11) = 2 \cdot 0.4562 = 0.9124$. Thus we will not reject the null hypothesis at any common significance level.

QUESTION 8

We can use a two-sample t-test with equal variances (as in the table with partial results). The estimate of the pooled variance is

$$s_p^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2}$$

$$= \frac{(65 - 1)1.344281^2 + (65 - 1)1.686032^2}{65 + 65 - 2} = 2.324898.$$

The test statistic is

$$Z = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{s_p^2}{n_x} + \frac{s_p^2}{n_y}}} = \frac{7.039558 - 6.395729}{\sqrt{\frac{2.324898}{65} + \frac{2.324898}{65}}} = 2.41.$$

The question does not specify the significance level, but because the managers have considered 95% confidence intervals, one alternative is to use a 5% significance level and a two-sided alternative. The critical value in the t-distribution with $65 + 65 - 2 = 128$ degrees of freedom is not tabulated. We have either $t_{0.025, 100} = 1.984$ or $Z_{0.025} = 1.960$, and in either case we reject the null hypothesis that the mean costs are equal.

Thus, the difference in means is statistically significant at the 5% significance level.

QUESTION 9

a. If a participant selects a bowl by a simple random draw, the probability of selecting the bowl with the hidden phone is $p = 1/4$. This can be used as the null hypothesis: $H_0 : p = 1/4$. The alternative hypothesis is then $H_1 : p > 1/4$.

We can regard each participant as a Bernoulli trial where the probability of success (correctly identifying the bowl with the phone) is $1/4$. The number of successes for all $n = 64$ participants will then have a binomial distribution. From the sample can obtain the sample proportion, \hat{p} . Given H_0 , the sample proportion will be approximately normally distributed because $n \cdot p(1-p) = 64 \cdot 0.25 \cdot 0.75 = 12 > 5$.

Choosing a 5% significance level, the critical value for the test statistic

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{\hat{p} - 0.25}{\sqrt{\frac{0.25(1-0.25)}{64}}}$$

is 1.645. This means we will reject H_0 if

$$\hat{p} > 0.25 + 1.645 \sqrt{\frac{0.25(1-0.25)}{64}} = 0.3561.$$

Multiplying with the sample size, we get $0.3561 \cdot 64 = 22.7$, thus we will reject H_0 if 22 or more participants identify the correct bowl.

b. It would be better to use four bowls than two. One way to see this is to consider the excess number of participants needed to reject the null hypothesis.

First, if we use two bowls, the hypotheses become $H_0 : p = 1/2$ and $H_1 : p > 1/2$, and we will reject H_0 if

$$\hat{p} > 0.5 + 1.645 \sqrt{\frac{0.5(1-0.5)}{64}} = 0.6028.$$

Multiplying with the sample size, we get $0.6028 \cdot 64 = 38.58$, so we will reject H_0 if 39 or more participants identify the correct bowl.

Second, the expected number of successes when $p = 0.5$ is $0.5 \cdot 64 = 32$, implying that we need $39 - 32 = 7$ participants more than expected (under H_0) in order to reject H_0 . When $p = 0.25$ is $0.25 \cdot 64 = 16$, implying that we need $22 - 16 = 6$ participants more than expected in order to reject H_0 . This difference is mainly driven by the fact that the $Var(\hat{p}) = p(1-p)/n$ has its maximum value for $p = 0.5$.¹

¹An alternative, less formal argument is to consider a situation with very many bowls, say 1000. Under H_0 it would then be unlikely for even a few, say 2, participants to identify the correct bowl.