

**EXAM IN HMET4100, 7 DECEMBER 2020:
SUGGESTED SOLUTIONS AND COMMENTS**

VERSION INTENDED FOR CANDIDATES

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INTRODUCTION

The exam was arranged digitally on the Inspera platform and consisted of 9 questions. Four of the questions were multiple choice questions where the candidates should choose one out of a set of alternative answers. The remaining questions were in “essay” format and the answers had to be written as text.

The maximum score for each question was 10 points and the minimum was 0 points. In total, the maximum score was 90 points.

The following relation between grades (A-F) and points [0-90] was used: A: [81-90], B: [71-80], C: [55-70], D: [41-54], E: [30-40], F: [0-29].

Explanation of grades. After the grades are published, the candidates will have access to their individual scores by logging on to Inspera. The present suggested solution, which will be distributed to the candidates, explains the general principles for how scores and grades were awarded.

Appeal against grades. Appeals against grades must be submitted following the procedure described on the course web-page (that is, not submitted to the course coordinator).

Informal feedback. Candidates who wish to receive additional feedback may ask the course coordinator for an informal talk (phone call or personal meeting). The candidates must then email the course coordinator 2-3 suggested times. These suggested times should be within normal office hours (0900h-1700h), with at least two workdays notice, and within the period 11-22 January 2021.

SUGGESTED SOLUTIONS

QUESTION 1

$$P(X = 9) = \frac{e^{-14.85} 14.85^9}{9!} = \underline{\underline{0.0417}}$$

QUESTION 2

$$M.E. = t_{n-1, \alpha/2} \frac{s}{\sqrt{n}} = 3.012 \frac{\sqrt{12.3}}{\sqrt{14}} = \underline{\underline{2.82}}$$

QUESTION 3

$P(Z < a) \approx 0.15$ for $a = -1.04$. Let $X_0 = -1.04 \cdot 15.4 + 298 \approx 282$. Then, $P(X < 282) \approx 0.15$.

QUESTION 4

The test statistic is

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{50.89504 - 50}{10/\sqrt{400}} = 1.79.$$

The p-value is $P(|Z| > 1.79) = P(Z < -1.79) + P(Z > 1.79) = 0.0367 + 0.0367 = \underline{0.0734}$.

QUESTION 5

$$P(5 < X \leq 8) = P(6) + P(7) + P(8) = 0.0803 + 0.1721 + 0.2581 = \underline{0.5105}$$

The probabilities can be obtained from the binomial table: define $Y = n - X$ as the number of failures and $q = 1 - p = 0.25$ as the probability of failure in each trial.

QUESTION 6

a)

$$\sigma_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}} = \sqrt{\frac{0.10(1-0.10)}{100}} = 0.030$$

b)

$$\sigma_{\hat{p}} = 0.046$$

c) The margin of error is

$$ME = 1.96\sqrt{\frac{P(1-P)}{n}}.$$

Taking the square and rearranging, we find the sample size needed to obtain a 95% confidence interval,

$$n = \frac{P(1-P) \cdot 1.96^2}{ME^2}.$$

The sample size is a quadratic function of P with maximum for $P = 0.5$ (ME is considered constant). Thus, the largest possible sample size needed is for $P = 0.5$. Note that the formula for the confidence interval relies on the normal approximation to the binomial distribution, that is $nP(1-P) > 5$ should be satisfied.

QUESTION 7

Let subscripts v and c denote the vaccine group and the control group, respectively. Null hypothesis: $H_0 : p_v = p_c$. Alternative hypothesis $H_1 : p_v \neq p_c$

Pooled estimator

$$\hat{p}_0 = \frac{n_v \hat{p}_v + n_c \hat{p}_c}{n_v + n_c} = \frac{34035 \cdot \frac{12}{34035} + 34003 \cdot \frac{15}{34003}}{34035 + 34003} = 3.968 \cdot 10^{-4} = 0.0003968$$

Test statistic

$$\begin{aligned} Z &= \frac{\hat{p}_v - \hat{p}_c}{\sqrt{\frac{\hat{p}_0(1-\hat{p}_0)}{n_x} + \frac{\hat{p}_0(1-\hat{p}_0)}{n_y}}} \\ &= \frac{\frac{12}{34035} - \frac{15}{34003}}{\sqrt{\frac{0.0003968(1-0.0003968)}{34035} + \frac{0.0003968(1-0.0003968)}{34003}}} = -0.58 \end{aligned}$$

The p-value is $P(Z < -0.58) + P(Z > 0.58) = 0.2810 + 0.2810 = 0.562$, thus, we will not reject the null hypothesis for any common significance level. The result indicates that intussusception occurs equally frequent in the two groups.

The test is based on the assumption that the test statistic is normally distributed. This is a good approximation when $nP(1-P) > 5$. In this case we have $34035 \cdot \frac{12}{34035} (1 - \frac{12}{34035}) = 12.0$ for the vaccination group and $34003 \cdot \frac{15}{34003} (1 - \frac{15}{34003}) = 15.0$ for the control group, so the normality assumption seems reasonable. (Another and more basic assumption is that the participants are randomly assigned either to the vaccination group or the control group.)

QUESTION 8

We perform a Mann-Whitney test.¹ The null hypothesis is that the median waiting times are the same at both health centers, while the alternative hypothesis is that the median waiting times are different.

$$U = 17 \cdot 18 + \frac{17(17+1)}{2} - 258 = 201,$$

$E(U) = 17 \cdot 18/2 = 153$, and $Var(U) = 17 \cdot 18(17 + 18 + 1)/12 = 918$. Observed test statistic:

$$Z = \frac{201 - 153}{\sqrt{918}} = 1.58.$$

The critical value for a two-sided test at the 5% significance level is 1.96, thus we do not reject the null hypothesis.

¹The Wilcoxon Rank Sum test will yield an identical end result, but the intermediate calculations are different: $T = 258$, $E(T) = 306$, $Var(T) = 918$, and $Z = -1.58$.

QUESTION 9

a) The null hypothesis is that the median waiting time is the same for all four centers, while the alternative hypothesis is that at least one median is different from the others. We have $K = 4$ groups and the critical value is $\chi^2_{4-1,0.05} = 7.815$. The observed test statistic is greater than the critical values, so we reject the null hypothesis and accept the alternative hypothesis.

b) The graph shows four box plots, one for each center. For each center, the median is the dark vertical line inside the box. The smallest value is for A (slightly less than 2), while the largest value is for B (approximately 4.5). The medians for centers C and D are between those two values.

If a population is normally distributed, we would expect the corresponding sample distribution to be normal if we have a large sample. If the distribution is normal, we would expect the median to be located near the midpoint between the lower quartile (left side of the box) and the upper quartile (right side of the box), and we would expect that the left and right whiskers to have approximately the same length.

The graph for A seems to be right-skewed (both box and whiskers), while B is consistent with a normal distribution.

c) (Several answers are possible.) Both the Mann-Whitney (MW) and Kruskal-Wallis (KW) tests are non-parametric test that are valid in small samples from populations that are not necessarily normally distributed. The MW test is valid for comparison of two independent groups. The KW test is an extension of the Mann-Whitney test that is intended for comparing three or more groups.

In general, it is better to use KW if we have more than two groups, because it takes all available information into account. This can be seen in the current example where MW fails to reject H_0 for centers A and D, which have the smallest and largest medians among the four groups, while the KW allows us to reject the null hypothesis.

If MW is used repeatedly to test all possible pairs (AB, AC, AD, BC, BD, and CD) the probability of making a type I error would be higher than indicated by the chosen significance level (confer chapter 15.2, "multiple comparison between subgroup means" for an analogue from one-way ANOVA).