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**EXAM IN HMET4100, 12 DECEMBER 2022:
SUGGESTED SOLUTIONS AND COMMENTS**

Knut R. Wangen
k.r.wangen@medisin.uio.no

INTRODUCTION

The exam was arranged digitally on the Inspera platform and consisted of 9 questions. The 3 first questions were multiple choice questions where the candidates should choose one out of a set of alternative answers. The remaining questions were in "essay" format and the answers had to be written as text.

The multiple choice questions were assessed automatically, while the "essay" questions were assessed manually. The maximum score for each question was 10 points and the minimum was 0 points. In total, the maximum score was 90 points.

The following relation between grades (A-F) and points [0-90] was used: A: [80-90], B: [70-79], C: [55-69], D: [40-54], E: [30-39], F: [0-29].

Explanation of grades. After the grades are published, the candidates will have access to their individual scores by logging on to Inspera. The present suggested solution, which will be distributed to the candidates, explains the general principles for how scores and grades were awarded.

Appeal against grades. Appeals against grades must be submitted following the procedure described on the course web-page (that is, not submitted to the course coordinator).

Informal feedback. A candidate who wishes to receive additional feedback may ask the course coordinator for an informal talk (phone call or Zoom meeting). The candidate must then email the course coordinator 2-3 suggested times with at least two workdays notice. The suggested times should be within normal office hours (0900h-1700h) in the period 5-13 January 2023.

QUESTION 1

$$P(|Y| > 1.356) = P(Y > 1.356) + P(Y < -1.356) = 2 \cdot P(Y > 1.356) = 2 \cdot 0.10 = \underline{\underline{0.20}}.$$

QUESTION 2

Alternative 1:

$$P(X = 3) = \frac{4!}{3!(4-3)!} \cdot 0.7^3 \cdot (1-0.7)^{(4-3)} = 0.4116, P(X = 4) = 0.7^4 = 0.2401, \\ \text{and } P(X \geq 3) = P(X = 3) + P(X = 4) = \underline{\underline{0.652}}$$

Alternative 2:

Define Y as the number of failures (X is the number of successes). The probability of failure in each Bernoulli trial is $q = 1 - 0.7 = 0.3$, so that Y is binomially distributed with $n = 4$ and $q = 0.3$. The values $X = 3$ and $X = 4$ correspond to the values $Y = 1$ and $Y = 0$, respectively. From the binomial probability table, we find $P(Y = 1) = 0.4116$ and $P(Y = 0) = 0.2401$, so that $P(X \geq 3) = P(X = 3) + P(X = 4) = P(Y = 1) + P(Y = 0) = \underline{\underline{0.652}}$.

QUESTION 3

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}; \quad 0.087 \pm 1.96 \sqrt{\frac{0.087(1-0.087)}{5387}}; \quad \underline{\underline{(0.079, 0.095)}}.$$

QUESTION 4

$$P(S \geq 14) = 1 - P(S \leq 13) = 1 - 0.942 = \underline{\underline{0.058}}.$$

A positive difference indicates that medication A is better than medication B (higher HRQOL). The p-value is 5.8%, implying that we can reject the null hypothesis at the 10% significance level but not at the 5% level. Some may say that the evidence gives weak support for the alternative hypothesis, that is, A is better than B.

If we had used a two-sided alternative hypothesis, the p-value would have been $2 \cdot 5.8\% = 11.6\%$, implying that we would have kept the null hypothesis also at the 10% significance level.

QUESTION 5

(Several variants are possible.) $H_0 : \mu_x - \mu_y = 0$ versus $H_1 : \mu_x - \mu_y \neq 0$. Two means, independent samples, unknown population variances assumed to be equal. The pooled variance is:

$$s_p^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2} \\ = \frac{(122 - 1)50350^2 + (128 - 1)44732^2}{122 + 128 - 2} = 2261575420.$$

The observed test statistic is:

$$T_{obs} = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_p^2}{n_x} + \frac{s_p^2}{n_y}}} = \frac{82836 - 90149}{\sqrt{2261575420} \cdot \sqrt{\frac{1}{122} + \frac{1}{128}}} = -1.215.$$

The critical value for the number of degrees of freedom ($122 + 128 - 2 = 248$) is not tabulated. If an $\alpha = 5\%$ significance level is chosen, the critical value must be between 1.960 (infinite number of degrees of freedom) and

1.984 (100 degrees of freedom). In either case, we do not reject H_0 because $|T_{obs}| = 1.215$ is less than the critical value.

The test is based on the assumptions that the populations are normally distributed. Due to the central limit theorem, the relatively large samples makes it reasonable to assume that the sample means are approximately normally distributed, even if the populations should not be normally distributed.

QUESTION 6

$$\bar{x} \pm t_{n-1, \alpha/2} \frac{s^2}{\sqrt{n}}; \quad 102.5502 \pm 2.750 \frac{14.08907^2}{\sqrt{31}}; \quad \underline{\underline{(95.6, 109.5)}}.$$

QUESTION 7

$$\bar{x}_{critical} = \mu_0 + z_{\alpha} \sigma / \sqrt{n} = 91.2 + 2.326 \cdot \sqrt{41.6} / \sqrt{168} = \underline{\underline{92.4}}.$$

QUESTION 8

$$W = \frac{12}{n(n+1)} \sum_{i=1}^k \frac{R_i^2}{n_i} - 3(n+1) =$$

$$\frac{12}{51(51+1)} \left(\frac{410^2}{18} + \frac{568^2}{17} + \frac{348^2}{16} \right) - 3(51+1) = \underline{\underline{6.379}}.$$

The critical values is $5.991 < W$, so we reject the null hypothesis.

QUESTION 9

$E(T) = \frac{n(n+1)}{4} = \frac{25}{25+1} = 162.5$, $Var(T) = \frac{n(n+1)(2n+1)}{24} = 1381.25$, $T = \min(T_+, T_-) = \min(85, 240) = 85$, and $Z_{obs} = \frac{85-162.5}{\sqrt{1381.25}} = -2.09$. Then, $p = 2 \cdot P(Z < -2.09) = 2 \cdot 0.0183 = \underline{\underline{0.037}}$.