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**EXAM IN HMET4101, DECEMBER 1, 2025:
SUGGESTED SOLUTIONS AND COMMENTS**

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INTRODUCTION

The exam was arranged digitally on the Inspera platform and consisted of 10 questions. The first two questions were multiple-choice questions in which the candidates should choose one of several alternative answers. The remaining questions were in an "essay" format and the answers had to be written as text.

The multiple-choice questions were automatically assessed, while the "essay" questions were manually assessed. The maximum scores per question ranged from 8 to 25 points. In total, the maximum score was 120 points.

The following relation between grades (A-F) and points [0-120] was used: A: [107–120], B: [91–106], C: [73–90], D: [56–72], E: [45–55], F: [0–44].

Explanation of grades. After the grades are published, the candidates will have access to their individual scores by logging into Inspera. The suggested solution presented here, which will be distributed to the candidates, explains the general principles for how the scores and grades were awarded. A candidate's detailed scores on the exam questions and the general suggested solution constitute the formal explanation of the grade, and no further formal explanations will be given.

Appeal against grades. Appeals against grades must be submitted following the procedure described on the course website (that is, not submitted to the course coordinator).

Informal feedback. A candidate who wishes to receive additional feedback can ask the course coordinator for an informal talk (say by phone or Zoom). The candidate must then email the course coordinator 2-3 suggested times with at least two workdays notice. The suggested times should be within normal office hours (09:00-17:00) in the period between December 22, 2025 and January 9, 2026.

QUESTION 1 (10P)

$$\begin{aligned}
P(X > 12) &= P\left(\frac{X - \mu}{\sigma} > \frac{12 - \mu}{\sigma}\right) \\
&= P\left(Z > \frac{12 - 9.2}{2.1}\right) = P(Z > 1.33) \\
&= 1 - P(Z < 1.33) = 1 - 0.9082 \approx \underline{\underline{0.092}}.
\end{aligned}$$

QUESTION 2 (10P)

For a two-sided test, the p-value is $p = 2 \cdot P(Z > |z|) = 2P(Z > 2.11)$. From the standard normal table, $P(Z > 2.11) = 1 - P(Z < 2.11) = 1 - 0.9826 = 0.0174$. Thus, $p = 2 \cdot 0.0174 = 0.0348 \approx \underline{\underline{0.035}}$.

QUESTION 3 (10P)

Let X be the number of vaccinated employees. We have $X \sim \text{Bin}(n = 20, p = 0.60)$, but the binomial tables provided list probabilities only for $p \leq 0.50$. We therefore define $Y = 20 - X$, the number of non-vaccinated employees. Then $Y \sim \text{Bin}(20, q)$ with $q = 1 - p = 0.40$.

a)

$$P(X = 12) = P(Y = 8) = 0.1797.$$

The probability after the last equality is taken from the table for individual binomial probabilities.

b)

$$\begin{aligned}
P(10 \leq X \leq 14) &= P(X = 10) + \dots + P(X = 14) \\
&= P(Y = 10) + \dots + P(Y = 6) = P(Y \leq 10) - P(Y \leq 5) \\
&= 0.872 - 0.126 = 0.746.
\end{aligned}$$

The probabilities in the last line are taken from the table for cumulative binomial probabilities.

The involved probabilities obtained from Desmos are reported in Figure 1.

QUESTION 4 (10P)

Using the "worst case" $p = 0.50$:

$$n = \frac{(Z_{0.025})^2 \cdot p(1 - p)}{ME^2} = \frac{1.96^2 \cdot 0.25}{0.10^2} \approx 96.04$$

Thus, a sample size of $n = 97$ is therefore required.

Effect of knowing last year's uptake was 60%:

$$n = \frac{1.96^2 (0.60)(1 - 0.60)}{0.10^2} \approx 92.16.$$

Thus, prior knowledge of $p = 0.60$ leads to a slightly smaller required sample size (93 instead of 97 after rounding up), because $p(1 - p)$ attains its maximum at $p = 0.5$.

FIGURE 1. Binomial probabilities, Question 3

$\frac{20!}{10!(20-10)!} 0.60^{10} (1-0.60)^{(20-10)}$	$= 0.1171415505$
$\frac{20!}{11!(20-11)!} 0.60^{11} (1-0.60)^{(20-11)}$	$= 0.159738478$
$\frac{20!}{12!(20-12)!} 0.60^{12} (1-0.60)^{(20-12)}$	$= 0.1797057878$
$\frac{20!}{13!(20-13)!} 0.60^{13} (1-0.60)^{(20-13)}$	$= 0.1658822656$
$\frac{20!}{14!(20-14)!} 0.60^{14} (1-0.60)^{(20-14)}$	$= 0.1244116992$
$0.1171 + 0.1597 + 0.1797 + 0.1659 + 0.1244$	$= 0.7468$ 

QUESTION 5 (10P)

(Other solutions are possible.) We can assume that the two samples are independent, and that the populations have normal distributions with unknown but equal variances.

We let subscripts A and B denote values from Clinics A and B, respectively.

$$(\bar{x}_A - \bar{x}_B) \pm t_{\alpha/2, n_A + n_B - 2} \cdot \sqrt{\frac{s_p^2}{n_A} + \frac{s_p^2}{n_B}},$$

where the pooled variance is

$$\begin{aligned} s_p^2 &= \frac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{n_A + n_B - 2} \\ &= \frac{(18 - 1)1.6^2 + (16 - 1)1.9^2}{18 + 16 - 2} = 3.0521875 \end{aligned}$$

Inserting values we obtain

$$(4.8 - 5.6) \pm 2.042 \cdot \sqrt{\frac{3.0521875}{18} + \frac{3.0521875}{16}} \rightarrow (-2.03, 0.43),$$

where $t_{0.025, 32} \approx t_{0.025, 30} = 2.042$ has been used as an approximation.

The interval includes 0, so the data do not provide clear evidence of a difference in mean waiting times between the two clinics.

QUESTION 6 (10P)

(Other solutions are possible.) Let p_x and p_y denote the vaccination proportions for nurses and auxiliary nurses, respectively. We state the following hypotheses, $H_0 : p_x \leq p_y \iff p_x - p_y \leq 0$ and $H_1 : p_x > p_y \iff p_x - p_y > 0$, and choose a $\alpha = 5\%$ significance level.

We use the pooled sample proportion

$$\hat{p} = \frac{n_x \hat{p}_x + n_y \hat{p}_y}{n_x + n_y} = \frac{80 \cdot \frac{53}{80} + 65 \cdot \frac{33}{65}}{80 + 65} = \frac{53 + 33}{80 + 65} = 0.5931034483$$

The following test statistic

$$Z = \frac{\hat{p}_x - \hat{p}_y}{\sqrt{\left(\frac{\hat{p}_0(1-\hat{p}_0)}{n_x} + \frac{\hat{p}_0(1-\hat{p}_0)}{n_y}\right)}}$$

is approximately standard normal, and we will reject the null hypothesis for values larger than the critical value $Z_{crit} = 1.645$.

Inserting values, we find

$$Z_{obs} = \frac{\frac{53}{80} - \frac{33}{65}}{\sqrt{0.5931034483(1 - 0.5931034483)}\sqrt{\left(\frac{1}{80} + \frac{1}{65}\right)}} = 1.887.$$

Given that $Z_{obs} = 1.887 > Z_{crit} = 1.645$, we reject the null hypothesis and accept the alternative hypothesis at the 5% significance level. From this we can conclude that, yes, the data support the belief that nurses had higher vaccination uptake than auxiliary nurses.

QUESTION 7 (25P)

There is a significant effect of gender, since the p-value in the output is 1.7% (1p). On average, males incur 1626 kroner lower costs than females. (1p)

A 95% confidence interval for the Age coefficient is given by (assuming 100 d.f.): $170.23 \pm 1.98 \cdot 64.02 \rightarrow (43, 297)$. (5p)

Predicted cost of a 100 year patient: $intercept \cdot age = 1642 + 170 \cdot 100 = 18642$ kroner. (2p) The problem with predicting the cost for 100-year-olds, may be that there are not many 100-year-olds in the data. Hence the prediction may be inaccurate. (2p)

We are assuming that age has a linear effect on cost, that is, an increase from 30 to 40 year in age has the same effect on costs as an increase from 70 to 80 years (2p). Whether this assumption is ok can be checked in a scatter plot. (2p)

Mean cost of Treatment 3: $Intercept + treatment2 = 14733 - 3999 = 10734$ kroner (2p). The intercept would correspond to the mean cost for patients on Treatment 1, as this is the reference category for Treatment (no indicator variable for Treatment 1). (3p)

We may compare the multiple R-square values for the regressions of each independent variable in the output. We see that Treatment explain 32% of the variation in Cost, whereas Gender and Age only explain 4.7% and 5.6%, respectively. Hence, Treatment seems to be the most important variable. (5p)

QUESTION 8 (15P)

Cost of a 30 year old male using Treatment 2: $4381 - 85 \cdot 1 + 163 \cdot 30 - 4379 \cdot 1 = 4807$ kr
 Cost of a 47 year old female using Treatment 3: $4381 - 85 \cdot 0 + 163 \cdot 47 - 4059 \cdot 1 = 7983$
 Hence, the female has 3176 kr higher costs. (5p)

The coefficient for gender means that males on average have 85 kr lower costs than females (non-significant). However, this is adjusted for Age and Treatment, hence conditional on comparing two patients with the same Age and Treatment. The estimate in the previous exercise was unconditional, i.e. did not take into account that Age and Treatment distributions may differ between the genders. (5p)

The histogram of the residuals should have the bell-shape of a normal distribution. It looks fine. The scatter plot should look like an irregular cloud, without any patterns. If it is funnel-shaped, it could indicate heteroscedasticity (e.g. the higher the values of the dependent variable, the greater the variation in the values). Few points are also outside the boundaries ± 2 (these are standardized residuals, so larger positive or negative values indicate outliers). Overall, it looks fairly ok. (5p)

QUESTION 9 (12P)

Treatment is a confounder of gender, as the regression coefficients for Treatment 2 and 3 are fairly constant between the simple and multiple analyses. Hence, the fact that treatment is differently distributed for males and females causes the coefficient for gender to change (5p).

Females appear more costly in the simple analyses simply because more of them stay at the urban hospital, where they use the expensive treatment. When both treatment and gender is in the model, you remove the hidden effect of treatment from gender (or, more precisely, you control for the fact that treatment is differently distributed in males and females) and are left with a more unbiased gender effect. Hence, gender in itself does not seem to be important. (7p)

QUESTION 10 (8P)

An interaction between gender and treatment would mean that the difference between genders is not constant across the treatments, or that the cost difference between treatments is not the same for males and females. (4p)

From the explanation, it would be surprising if there was an interaction, as this would mean that the medication was more expensive for e.g. males than females. Such an interaction is not expected. One exception would be if e.g. males actually needed more medication, thus resulting in higher costs. (4p)